Collusion in Large Contests

Phil Brookins[1] Paan Jindapon[2]

[1]Darla Moore School of Business
University of South Carolina

[2]Culverhouse College of Business
University of Alabama

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Background

- Cooperation in infinitely repeated games
- The folk theorem
- Grim trigger strategy
- Prisoners' dilemma, Cournot/Bertrand competition, team production

Our Focus

- Cooperation in large games (multiple players)
- Winner-take-all contests (Tullock)

Theory

- Oligopoly: Green (JET 1980) and Lambson (JET 1984)
- Public good: Pecorino (AER 1998) and Pecorino (J Pub E 1999)

I. Background

Experiments with strategic complements and substitutes

- Chen and Gazzale (AER 2004)
- Potters and Suetens (REStud 2009)
- Mermer, Müller, and Suetens (JEBO 2021)

I. Background

Infinitely repeated public-good experiment with 4 players

- Lugovskyy, Puzzello, Sorensen, Walker, and Williams (GEB 2017)

Infinitely repeated contest experiment with 2 players

- Brookins, Ryvkin, and Smyth (EE 2021)
- Deck, Dorobiala, and Jindapon (JESA 2024)

II. Theory

- There are $n \geq 2$ players in a symmetric standard Tullock contest.
- The contest winner receives $v > 0$.
- In Nash equilibrium,

$$x_i^e = \left(\frac{n-1}{n^2}\right) v$$

and

$$\pi_i^e = \frac{v}{n^2}$$

for $i = 1, ..., n$.

II. Theory

- If all n firms collude by investing $cv < x_i^e$, then

$$x_i^c = cv$$

where $c < \frac{n-1}{n^2}$, and

$$\pi_i^c = \left(\frac{1}{n} - c\right) v$$

for $i = 1, ..., n$.

Grim Trigger strategy with group punishment

- I begin with investing cv, where $c < \frac{n-1}{n^2}$, in the contest.
- I'll keep investing cv as long as I observe that the total investment does not exceed ncv.
- If I observe that the total investment exceeds ncv, then I'll invest $\frac{n-1}{n^2}v$ forever after.

Such a coordination can be sustained if

$$\left(\frac{1}{1-\delta}\right)\pi_i^c \geq \pi_i^d + \left(\frac{\delta}{1-\delta}\right)\pi_i^e$$

where π_i^d is player i's defect payoff.

II. Theory

- In the defect period, suppose $x_j = cv$ for all $j \neq i$. Then,

$$\pi_i^d = \left[1 - \sqrt{c(n-1)}\right]^2 v.$$

- Let $\bar{\delta}$ be the minimum discount factor supporting the collusion with the level of effort cv. Then,

$$\bar{\delta}(c) = \frac{n\left(\sqrt{n-1} - n\sqrt{c}\right)^2}{n^2\left[1 - \sqrt{c(n-1)}\right]^2 - 1}.$$

- For $c = 0$ (most efficient),

$$\bar{\delta}(0) = \frac{n}{n+1}.$$

II. Theory

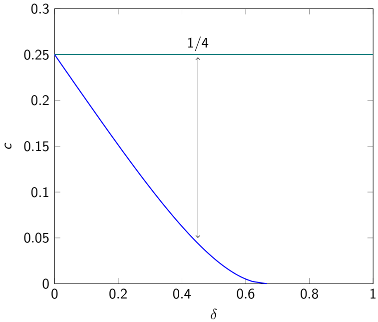
We find that

- $\bar{\delta}(c)$ is strictly decreasing in c
- $\lim_{c \to 0} \bar{\delta}(c) = \frac{n}{n+1}$
- $\lim_{c \to x_e} \bar{\delta}(c) = 0$

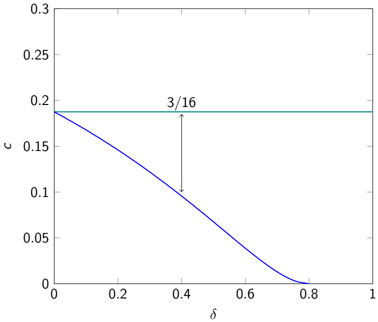
Therefore

- Given any $\delta \in (0, 1)$, there exists a value of c such that the collusion can be sustained.
- Given any $c \in [0, x^e)$, there exists a value of δ such that the collusion can be sustained.

II. Theory



(a) $n = 2$



(b) $n = 4$

III. Experiment

Design

- Standard Tullock Contest with 4 players‘
- 2 by 2:
  - Infinitely repeated game VS Finite number of contests
  - High δ VS Low δ
- TIDE Lab, University of Alabama

Parameters

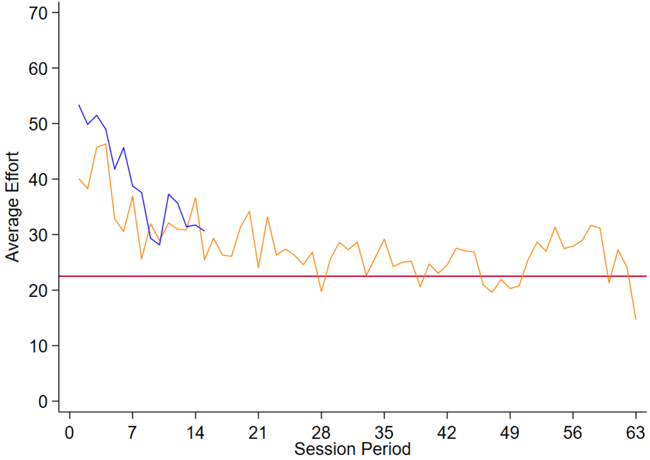
- Endowment: 120 ECU
- Prize: 120 ECU
- High $\delta = 0.8$
- Low $\delta = 0.5$
- Similar to Brookins, Ryvkin, and Smyth (2021) except
  - $n = 4$
  - Nash Equilibrium in stage game: 22.5 ECU

Treatments

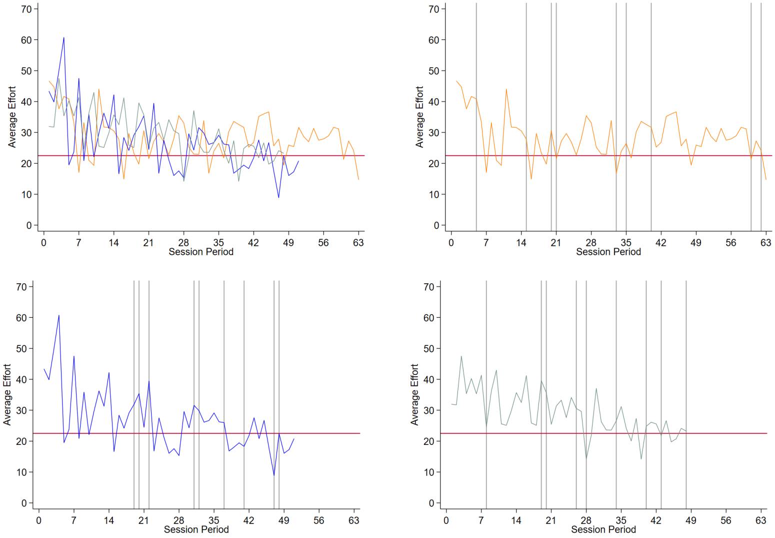
1. Infinite & High δ (10 supergames with $\delta = 0.8$):
   - 3 different sequences (63, 51, 48 periods)
   - 3 sessions (12, 16, 16 subjects)
   - 75 minutes, $20-$70 per subject
2. Infinite & Low δ (10 supergames with $\delta = 0.5$):
   - 3 different sequences (14, 15, 14 periods)
   - 3 sessions (16, 16, 16 subjects)
   - 45 minutes, $10-$20 per subject
3. Finite & High δ (50 periods):
   - Coming soon.
4. Finite & Low δ (20 periods):
   - Coming soon.

III. Experiment - Average effort by treatment

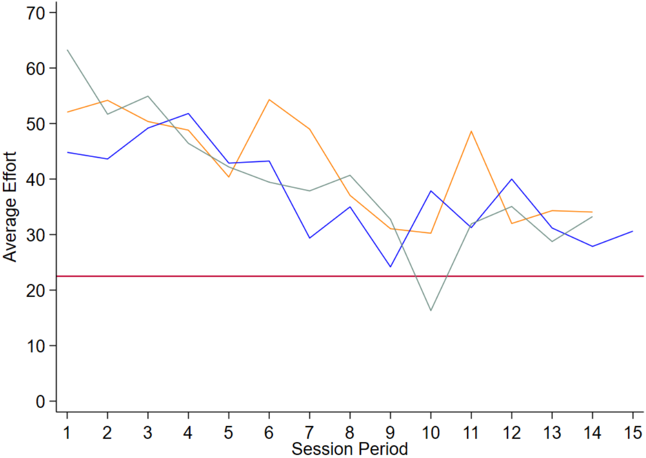
Data: all periods
orange: $\delta = 0.8$, blue: $\delta = 0.5$



III. Experiment - Average effort by session ($\delta = 0.8$)

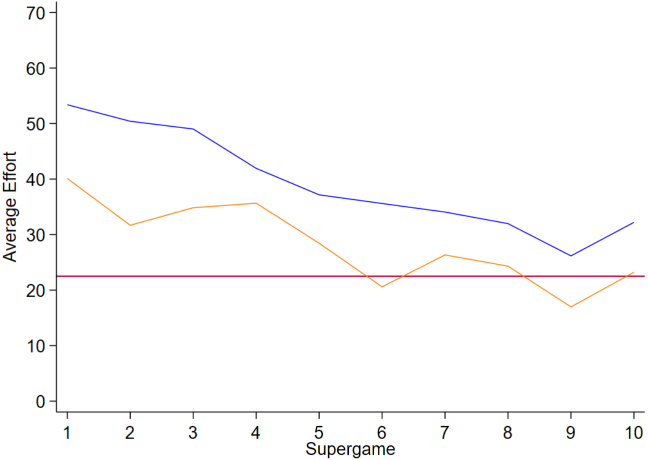


III. Experiment - Average effort by session ($\delta = 0.5$)



III. Experiment - Average effort by treatment

Data: first period of each supergame
orange: $\delta = 0.8$, blue: $\delta = 0.5$



III. Experiment - Learning by treatment

Dependent variable: Effort
Fixed effects: subjects
Data: all periods VS periods 1-14

	All Periods		**Periods 1 - 14**	
	$\delta = 0.8$	$\delta = 0.5$	$\delta = 0.8$	$\delta = 0.5$
Constant	39.473***	59.029***	47.552***	58.986***
	(1.496)	(3.471)	(3.931)	(3.579)
Period	-0.6994***	-3.8353***	-3.2312***	-3.7933***
	(0.1189)	(1.0359)	(1.2058)	(1.0975)
Period^2	0.0078***	0.1350**	0.1589**	0.1317*
	(0.0020)	(0.0655)	(0.0782)	(0.0712)
Number of:				
total observations	2,340	688	1,600	672
groups	44	48	160	48
observations per group	48/51/63	14/15/14	14	14

III. Experiment - Learning by treatment

Dependent variable: Effort
Fixed effects: subjects
Data: first period of each supergame

	$\delta = 0.8$	$\delta = 0.5$	$\delta = 0.8$	$\delta = 0.5$
Constant	40.395***	59.641***	39.670***	54.890***
	(3.261)	(3.672)	(2.486)	(2.413)
Period	-0.5523**	-4.1019***		
	(0.2402)	(1.1102)		
Period^2	0.0033	0.1419**		
	(0.0040)	(0.0714)		
Supergame			-2.0821***	-2.8535***
			(0.4006)	(0.3889)
Number of:				
total observations	440	480	440	480
groups	44	48	44	48
observations per group	10	10	10	10

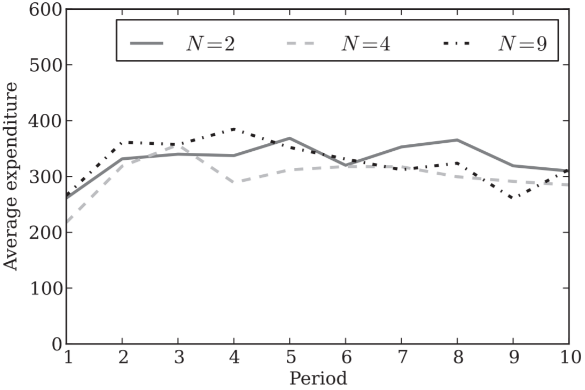
III. Experiment - Collusion ($\delta = 0.8$ VS $\delta = 0.5$)

Dependent variable: Effort
Method: OLS, SE clutered by subject
Data: all periods/periods 1-14/First period of each supergame

	All periods	Periods 1-14	First period of supergame
Constant	51.410***	59.631***	54.784***
	(3.590)	(4.622)	(3.801)
Period	-0.4813	-2.8382***	-0.0737
	(0.3068)	(0.8338)	(0.4449)
Period^2	0.0088**	0.1403**	0.0035
	(0.0039)	(0.0535)	(0.0054)
Supergame	-1.5727**	-1.5892**	-2.7799***
	(0.6469)	(0.7841)	(0.8256)
$\mathbb{1}[\delta = 0.8]$	-11.284***	-11.491**	-12.869**
	(5.362)	(5.400)	(6.128)
Observations	3,028	1,288	920
R-squared	0.0535	0.0433	0.0839

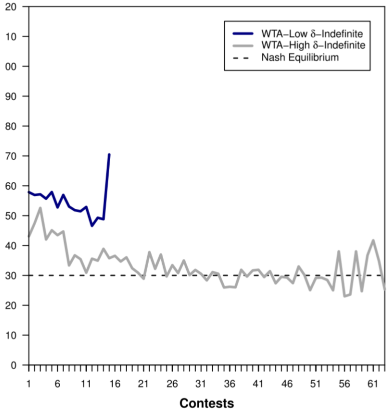
IV. Discussion

Lim, Matros, and Turocy (JEBO 2014)

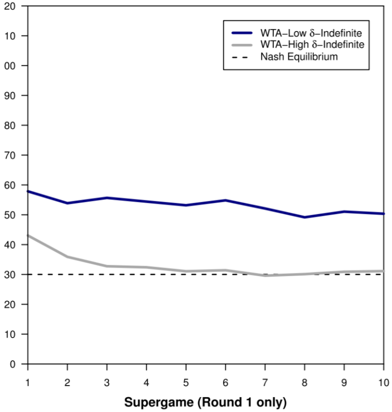


g. 3. Mean expenditure by period, aggregated across sessions, for each group siz

Brookins, Ryvkin, and Smyth (EE 2021)



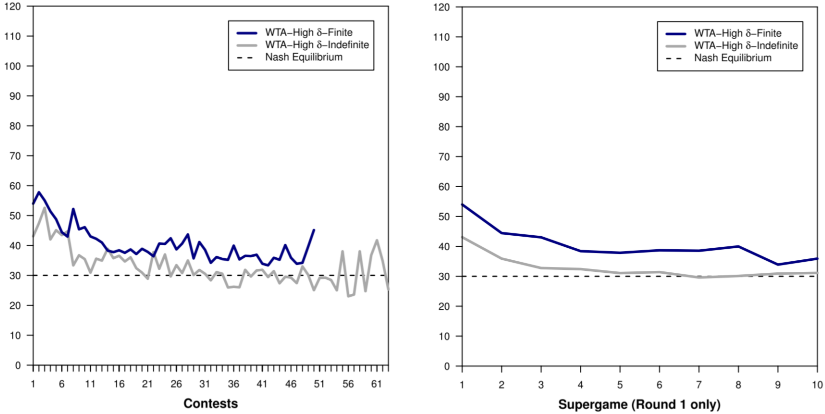
(a) WTA-δ Comparison



(c) WTA-δ Comparison

IV. Discussion

Brookins, Ryvkin, and Smyth (EE 2021)



(a) *WTA-Indefinite* Comparison (c) *WTA-Indefinite* Comparison

Thank You!

pjindapon@ua.edu