Bargaining over R&D Technology in Open Innovation

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Abstract

We theoretically model an R&D competition in which one of the firms can license out its advanced technology to its competitors. Each licensee pays the licensor a two-part tariff consisting of a fixed fee and a contingent fee. We use a Tullock contest as the underlying mechanism for determining the winner of the competition. Prior to the contest, firms are allowed to form a licensing coalition and bargain over the tariff. We find that a coalition always exists in equilibrium and the size of the coalition depends the bargaining power of each firm and the significance of the licensed technology.

Keywords: research & development; open innovation; bargaining; contest theory; technology licensing; coalition formation; network stability

JEL Classifications: L24; O36; O32; C72

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Instead of managing your IP to exclude rivals, manage your IP to profit from others' use of it.

Henry Chesbrough (2003)

1 Introduction

Chesbrough (2006) formally defines open innovation as the use of purposive inflows and outflows of knowledge to accelerate internal innovation and expand the markets for external use of innovation. Thus, there are two facets to open innovation: the *outside-in* aspect, where external technologies are brought into a firm's own innovation process, and the *inside-out* aspect, where a firm's technology is incorporated into other firms' innovation processes.

In this paper, we propose a theoretical framework to analyze open innovation among firms in a research and development (R&D) competition. We find that an open innovation coalition always exists in equilibrium whenever there is a firm with an advantage in R&D technology. However, the size of such a coalition and the structure of the corresponding license contract depend on the number of firms in the competition and the relative advantage of the technology shared within the coalition.

Our theoretical prediction that the technologically advanced firm will always form a coalition with at least one other firm with an inferior R&D technology is supported by empirical studies of open innovation in various industries. In his seminal books, Chesbrough (2003, 2006) uses case studies from Xerox, IBM, Intel, Lucent Technologies, Millennium Pharmaceuticals, Procter & Gamble, and Air Products and Chemicals to demonstrate how open innovation brought success to the R&D businesses of these firms. Other examples of technology licensing between rivals documented in the literature include IBM and Apple (Hagedoorn, Carayannis and Alexander, 2001), Nokia and Motorola (Dittrich and Duysters, 2007), Sony and Samsung (Gnyawali and Park, 2011), and German car manufacturers (Ili, Albers and Miller, 2010). In addition to the aforementioned specific case studies, other empirical studies provide a bigger picture of open innovation. For example, Cassiman and Veugelers (2006) use survey data from Belgium to show that internal R&D and external knowledge acquisition are complementary in innovation activities. Aschhoff and Schmidt (2008) use survey data from Germany to show that R&D cooperation with competitors leads to greater cost reductions that are attributable to innovative processes. Based on a Korean innovation survey, Lee, Park and Bae (2017) find evidence that adopting external technology through licensing-in does not always enhance innovative performance; the effectiveness of licensing-in varies with technological regimes.

For the framework of our analysis, we adopt a Tullock contest (Tullock, 1980) where there is only one winner and the probability of winning the contest for each firm is the ratio of the firm's R&D investment to the total R&D investment by all of the competing firms. Thus, the more R&D investment, the more likely to win the contest. However, the contest winner is not necessarily the firm that invests the most, that is, the contest winner-deciding process is not perfectly discriminating. Our analysis is the first to incorporate licensing in the canonical model of R&D competition. Specifically, we assume that one of the competing firms has an advantage in its R&D technology and this firm (i.e., the licensor) has an option to license out such a technology to some or all of the other firms (i.e., the licensees) before the contest begins.[1] Such a contract is nonexclusive so the licensor still has access to the technology, however, the licensees are not allowed to share or transfer the technology to any other firms.

We allow the licensor to choose any number of licensees to form a coalition and derive the optimal license contract between the licensor and licensees in the coalition. Such a license contract may consist of a fixed fee which is paid before the contest begins and a contingent fee which is paid only if the licensee wins the contest. We assume this two-part tariff structure to capture the two most common governance modes of technology alliance in the real world: a fixed compensation from a contractual agreement and a contingent compensation from an equity joint venture (Pisano, 1989; Oxley, 1999). Alternatively, the contingent component for the contract can be interpreted as a grant-back clause, which

[1]The R&D technology licensed and shared in our model is called generative appropriability in Ahuja, Lampert and Novelli (2013).

allows the licensor to reap additional benefit if its licensee wins the contest (Leone and Reichstein, 2012; Laursen et al., 2017).

Our model features both inside-out and outside-in aspects of open innovation. For the inside-out facet, the technologically advanced firm faces the licensing dilemma (Arora and Fosfuri, 2003) which is a trade-off between the additional revenue generated in the form of licensing payments and the erosion of (expected) profits because the licensees have become stronger competitors.In our setting where the contest outcome is uncertain (imperfectly discriminating), a contingent fee serves as a hedging instrument for the licensor since the licensor receives a portion of the contest prize from the licensee even when it is not the contest winner. For the outside-in facet, each licensee compares the benefit and cost of licensing in the R&D technology. While the incoming technology can increase the probability of winning the contest, the firm has to pay a fixed fee to the licensor irrespective of whether its innovation succeeds or not, and also has to give up a portion of the contest prize in the form of contingent license payment after its innovation succeeds.Given the tension between the licensor and licensee in terms of making a licensing contract, we find that there exists a combination of fixed and contingent fees that is acceptable to both the licensor and any number of licensees and, thus, a coalition of open innovation can be formed endogenously. An implication of such a coalition from our model is the ability to deter firms that are not in the coalition from competing. Arora (1997) and Arora, Fosfuri and Gambardella (2001) provide various business cases of firms using technology licensing to control competition and limit entry.

To derive the fixed and contingent fees to be chosen in a given coalition, we adopt two approaches: (1) assume that all the firms in the coalition have equal bargaining power and employ a Nash bargaining solution concept or (2) assume that the licensor has all the bargaining power and make a take-it-or-leave-it offer the licensees.[2] Based on the contractual outcomes for every possible coalition, we further consider an optimal coalition structure from the viewpoint of the licensor. More specifically, we model open innovation in R&D competition in three stages. In stage 1, a coalition is formed. In stage 2, the license

[2] See Sakakibara (2010) for an estimation of bargaining power between technology licensors and licensees in Japan.

fee is determined. In stage 3, the R&D contest begins.

We find that the equilibrium number of firms in the coalition depends on three factors: the total number of firms in the contest, the bargaining power of each firm in the coalition, and whether the technology of the advanced firm is radical. We define radical technology as the advanced firm's technology such that, if shared with at least one licensee, then all other firms (non-licensees) will drop out off the contest. If the total number of firms is four or fewer, then the advanced firm will include all other firms in the coalition, i.e., form a grand coalition. If the total number of firms is five or more and the advanced firm's technology is radical, then the size of the coalition depends on the license fee bargaining process. If the advanced firm is the only one with bargaining power, then it will form a grand coalition. If all firms in the coalition have equal bargaining power, then the advanced firm will include only one other firm in the coalition, i.e., form a two-firm coalition. We find that firms receive a higher (lower) expected profit than in status quo if they are in (not in) the coalition. This negative impact on firms outside the coalition is consistent with empirical finding in Oxley, Sampson and Silverman (2009).

We discuss the related literature in Section 2. We describe the three-stage contest in Section 3. Since we solve the game backward, we present equilibrium investment in the contest (stage 3), the license fee solution (stage 2), and the optimal licensing coalition (stage 1) in Sections 4, 5, and 6, respectively. We conclude in Section 7.

2 Related Literature

The analysis in this paper provides a bridge between two strands of literature, technology licensing mechanism and R&D network formation. To the best of our knowledge, this is the first theoretical paper that allows for endogenous network formation and knowledge sharing via licensing in a competition for innovation.

2.1 Technology licensing mechanism

The vast majority of the theoretical literature on technology licensing focuses on optimal fee structure or licensing mechanism in an oligopoly setting (see, for example, Katz and

Shapiro (1985, 1986) and Kamien and Tauman (1986)). While we analyze cost-reducing technology licensing among R&D firms in an innovation competition, the analyses under oligopoly that are closest to us are Sen and Tauman (2007) and Fan, Jun and Wolfstetter (2018). Similarly to our model, both papers assume that one of the competing firms has a cost-reducing technology and allow such a firm to choose the number of its licensees. While we use both Nash Bargaining and take-it-or-leave-it solutions and to derive the license fee in equilibrium, Sen and Tauman (2007) use the bids of auction winners and Fan, Jun and Wolfstetter (2018) let the firm with the technology make a take-it-or-leave-it offer. Fan, Jun and Wolfstetter (2018) find that the technology advanced firm will license its technology out to all other firms and Sen and Tauman (2007) obtain the same result in most cases. In contrast, we find that the technology advanced firm will make all other firms its licensees if the total number of firms is 4 or less. If the total number of firms is greater than 4, then the number of licensees depends on the impact of the technology.

Most fee structures studied in the literature are a fixed fee, a royalty rate, or a combination of both which is called a two-part tariff. In our model, we assume two-part tariff that consists of a fixed component which is paid up-front and a contingent component which is paid only if the licensee wins the competition. The contingency in the fee structure accommodates all parties involved in the contract; the licensees do not have to pay the licensor if they do not win the contest and the licensor has an opportunity to be rewarded (by the winning licensee) even when it does not win the contest. These effects are similar to grant-back clause which allows the licensor to reap additional benefits from the licensee's future innovation that stems from the licensor's knowledge. van Dijk (2000) and Choi (2002) introduce a grant-back clause to a licensing contract between two competing R&D firms. While van Dijk (2000) assumes a zero licensing fee, Choi (2002) allows the licensor to make a take-it-or-leave-it offer to the licensee. Both papers show a grant-back clause will reduce R&D investment and improve welfare of both firms.

An alternative interpretation of the contingent component of the licensing fee in our model is partial equity ownership (PEO), as analyzed in Ghosh and Morita (2017). In their model, the technology recipient partially loses its ownership (i.e., a portion of its profit) to the firm that provides a low-cost technology in production. While Ghosh and Morita (2017) impose the condition that the advanced firm can share its knowledge with only one of its competitors, we allow the the advanced firm to choose the number of technology recipients and, therefore, obtain PEO in multiple firms. Moreover, the contingent fee rate in our model, which is equivalent to the PEO parameter in Ghosh and Morita (2017), is determined endogenously using a Nash Bargaining solution.

d'Aspremont, Bhattacharya and Gérard-Varet (2000) analyze bargaining over interim knowledge, similar to technology used in R&D in this paper, but they focus on the amount of knowledge disclosed between two innovators, i.e., from the leading to the lagging firms. In a more closely related work, Clark and Konrad (2008) model a two-player two-stage R&D competition where the two firms compete in parallel Tullock's contests (to win patents for various innovative attributes) in stage 1 and then negotiate over the patents so that one of the firms has patents for all of the attributes in stage 2. Clark and Konrad (2008) use a Nash bargaining solution to derive each firm's share of the surplus from production. Since we focus on bargaining over technology used in an innovation competition (not in production of final products), the negotiations among all the firms in the coalition in our model occur before the contest begins. Similar Nash bargaining concept has been applied to derive licensing fees between an innovator and a producer (Spulber, 2016) and between two producers in a duopoly (Kishimoto and Muto, 2012).

2.2 R&D network formation

Goyal and Moraga-González (2001) is the first paper to formally analyze endogenous network formation for R&D cooperation in oligopoly (see, for example, d'Aspremont and Jacquemin (1988) and Kamien, Muller and Zang (1992)). Like Goyal and Moraga-González (2001), we let firms form a network of coalition in the first stage of the game. A key difference between the two models is that our network structure is bipartite because we focus on technology transfers between two types of firms (i.e., from a low-cost firm to a high-cost firm). While Goyal and Moraga-González (2001) show that a complete network is stable, we find that a complete bipartite network might not be stable when the number of firms is large and the transferred technology is radical.

Similarly to the patent race in Joshi (2008) and the Tullock contest in Grandjean, Tellone and Vergote (2017), the return from the R&D investment for each firm in our model is uncertain, and a greater R&D investment implies a greater probability of success. While firms in Joshi (2008) compete on behalf of their team, and each firm in the winning team shares an equal profit, firms in Grandjean, Tellone and Vergote (2017) and this paper compete individually, even though they are in a coalition.[3] Nonetheless, we find that a coalition may serve as a barrier to entry, because the high-cost firms that are not in a coalition will drop out of the competition. This result is comparable to those found in Marinucci and Vergote (2011) and Petrakis and Tsakas (2018).

3 The Model

There are $n \geq 3$ R&D firms taking part in a competition for innovation in which a fixed prize $R > 0$ (i.e., a patent right) is awarded to the winner of the competition.[4] Each firm makes a costly effort (i.e., an R&D investment) to improve the odds of winning the competition. We let $N \equiv \{1, 2, ..., n\}$ be the set of all R&D firms and denote firm i's effort by $x_i \geq 0$. We assume that firm i's marginal cost of effort is c_i, where $c_1 = 1$ and $c_i = c > 1$ for $i = 2, ..., n$. That is, firm 1 is the only low-cost firm and all other firms have a higher marginal cost of effort. We assume that all firms simultaneously choose their effort levels and each firm's probability of winning the competition follows the standard Tullock contest success function. Specifically, firm i's probability of winning is the ratio x_i/X, where $X = \sum_{j=1}^{n} x_j$. If $X = 0$, the winning probability of firm i is $1/n$ for all i.

We allow firms to interact in two stages prior to the contest. In stage 1, firm 1 forms a coalition with m other firms where $m \in \{0, 1, 2, ..., n-1\}$. Each of these m firms will be granted access to firm 1's low-cost technology so that their marginal cost of effort in

[3] Even though firms can form a coalition before they compete, the contests in Grandjean, Tellone and Vergote (2017) and in this paper are different from group contests where players compete on behalf of their group. See, for example, Baik and Shogren (1995), Sánchez-Pagés (2007), and Lee and Kim (2022).

[4] We omit the case $n = 2$ because the results are relatively less interesting; there are only two possibilities: a coalition exists or does not exist in equilibrium and when it does, there will be exactly two firms in the coalition and no other firms not in the coalition.

the contest becomes unity. In return, these technology recipients agree to pay firm 1 a fixed fee and a contingent fee that are proportional to R. Specifically, for ϕ and $\rho \in [0, 1]$, each of these firms pays firm 1 ϕR upon receiving the technology and ρR only if it wins the contest. In stage 2, the firms in the coalition choose ϕ and ρ through negotiation. We consider two scenarios in selecting ϕ and ρ: (1) all the firms in the coalition negotiate with equal bargaining power and (2) firm 1 is the only one with bargaining power. We derive the Nash bargaining (NB) solution in the first scenario and firm 1's optimal take-it-or-leave-it (TIOLI) offer in the latter.

We explain how the game proceeds in detail below.

3.1 Stage 1

The game begins with firm 1 forming a coalition with the objective of sharing its R&D technology with m other firms through a nonexclusive and nontransferable license agreement. Firm 1 is also allowed to remain in autarky, i.e., there is no coalition in the competition. Since firms $2, ..., n$ are identical, without loss of generality, we let the coalition only include firms $1, 2, ..., m+1$ so that firm 1's choice in stage 1 is the value of $m \in \{0, 1, ..., n-1\}$. We let C_m denote the set of firms $\{1, 2, ..., m+1\}$ in the coalition. Alternatively, we can say that firm 1 chooses a bipartite network g, i.e., a set of edges between firm 1 and other firms, from the set $\{\emptyset, \{12\}, \{12, 13\}, ...\{12, 13, ..., 1n\}\}$ because of the 1-to-1 correspondence between m and g. If $m = n-1$, then we say that firm 1 forms a grand coalition or a complete bipartite network.

3.2 Stage 2

After a value of m is chosen in stage 1, the licensor and licensees negotiate ϕ and ρ in stage 2. In the first scenario where all the firms in the coalition have equal bargaining power, they are assumed to employ the NB solution in choosing ϕ and ρ. Namely, they choose ϕ

and $\rho \in [0, 1]$ that maximize the Nash product

$$W(\phi, \rho|m) \equiv \Pi_{i=1}^{m+1}[\pi_i^e(\phi, \rho|m) - \pi_i^a] \tag{1}$$
$$\text{subject to } \pi_i^e(\phi, \rho|m) \geq \pi_i^a \text{ for all } i \in C_m,$$

where $\pi_i^e(\phi, \rho|m)$ denotes firm i's expected payoff in the equilibrium of the contest as a function of ϕ and ρ for the given $m \neq 0$, and π_i^a denotes firm i's disagreement payoff or the expected payoff in the equilibrium of the contest without a coalition (i.e., in autarky). In the other scenario where bargaining power is given to only firm 1, it is assumed to employ the TIOLI offer. Firm 1 then chooses ϕ and $\rho \in [0, 1]$, which maximizes its expected payoff

$$\pi_1^e(\phi, \rho|m) \tag{2}$$
$$\text{subject to } \pi_i^e(\phi, \rho|m) \geq \pi_i^a \text{ for all } i \in C_m.$$

3.3 Stage 3

Given the m, ϕ, and ρ chosen in the previous stages, all the firms choose their effort levels to win the contest. We let $\mathbf{x}_{-i}$ be a vector of effort levels chosen by all firms except i. Each firm's expected payoff in stage 3 is then given by

$$\pi_1(x_1|\mathbf{x}_{-1}) = \frac{x_1}{X}R - x_1 + m\phi R + \sum_{i=2}^{m+1} \frac{x_i}{X}\rho R, \tag{3}$$
$$\pi_i(x_i|\mathbf{x}_{-i}) = \frac{x_i}{X}(1 - \rho)R - x_i - \phi R, \tag{4}$$
$$\pi_j(x_j|\mathbf{x}_{-j}) = \frac{x_j}{X}R - cx_j \tag{5}$$

for $i = 2, ..., m + 1$ and $j = m + 2, ..., n$. Note that firm i is a technology recipient whose marginal cost is reduced to 1, whereas firm j's marginal cost remains c. We assume that all of the above is common knowledge among the firms and use a subgame perfect equilibrium as our solution concept. Figure 1 summarizes how the game proceeds.

Solving the game backwards, we first derive equilibrium contest outcomes in stage 3 in the next section. Then, we derive the license fees to be chosen within the coalition in stage

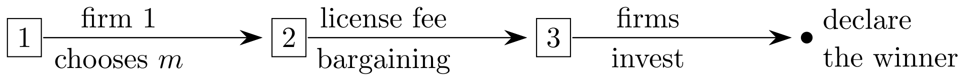


Figure 1: The timeline of the game

2, and finally, get the optimal value of m for the coalition in Sections 5 and 6, respectively.

4 Equilibrium Effort in Stage 3

We let X^a denote the total investment in autarky ($m = 0$) equilibrium and let $X^e(\rho|m)$ denote the total investment in equilibrium for the given ϕ and $\rho \in [0, 1]$ in stage 2 and given $m \in \{1, ..., n-1\}$ in stage 1. Note that the fixed component of the license fee (ϕ) does not affect the total investment in equilibrium.

Proposition 1 *In autarky, all firms are active in equilibrium and the total investment in equilibrium is given by*

$$X^a = \left[\frac{n-1}{(n-1)c+1}\right] R. \tag{6}$$

Suppose that a coalition $C_m = \{1, 2, ..., m+1\}$ is formed in stage 1.

(i) If $m = n-1$, or $m < n-1$ and $\frac{mc-m-1}{mc} < \rho < \frac{(n-m-1)(c-1)+1}{(n-m-1)c+1}$, then all firms are active in equilibrium and

$$X^e(\rho|m) = \left[\frac{m(1-\rho) + (n-m-1)}{m + (n-m-1)c + 1}\right] R. \tag{7}$$

(ii) If $m < n-1$ and $\rho \le \frac{mc-m-1}{mc}$, then only firms in C_m are active in equilibrium and

$$X^e(\rho|m) = \left[\frac{m(1-\rho)}{m+1}\right] R. \tag{8}$$

(iii) If $m < n-1$ and $\rho \ge \frac{(n-m-1)(c-1)+1}{(n-m-1)c+1}$, then only firm 1 and the firms not in C_m are active in equilibrium and

$$X^e(\rho|m) = \left[\frac{n-m-1}{(n-m-1)c+1}\right] R. \tag{9}$$

Proof. See Appendix A.1.

Two comparative statics results of Proposition 1 are presented in Remarks 1 and 2.

Remark 1 *Consider $X^e(\rho|m)$ derived in Proposition 1.*

(i) For a given m, $X^e(\rho|m)$ is strictly decreasing (constant) in ρ if $\rho < (>)\ \frac{(n-m-1)(c-1)+1}{(n-m-1)c+1}$.

(ii) For a given ρ, $X^e(\rho|m)$ is strictly increasing (decreasing) in m if $\rho < (>)\ \frac{(n-1)(c-1)}{(n-1)c+1}$.

Proof.

(i) In case (i) of Proposition 1, $\frac{\partial X^e}{\partial \rho} = -\frac{m}{m+(n-m-1)c+1}R < 0$. In case (ii) of Proposition 1, $\frac{\partial X^e}{\partial \rho} = -\frac{m}{m+1}R < 0$. In case (iii) of Proposition 1, $\rho > \frac{(n-m-1)(c-1)+1}{(n-m-1)c+1}$ and $\frac{\partial X^e}{\partial \rho} = 0$.

(ii) In case (i) of Proposition 1, $\frac{\partial X^e}{\partial m} = \frac{(n-1)(c-1)-[(n-1)c+1]\rho}{[m+(n-m-1)c+1]^2}R \gtreqless 0$ if and only if $\rho \lesseqgtr \frac{(n-1)(c-1)}{(n-1)c+1}$. In case (ii) of Proposition 1, $\frac{\partial X^e}{\partial m} = \frac{1-\rho}{(m+1)^2}R > 0$. In case (iii) of Proposition 1, $\frac{\partial X^e}{\partial m} = -\frac{1}{[(n-m-1)c+1]^2}R < 0$. ■

Remark 1 suggests that, across all three cases discussed in Proposition 1, the total investment in equilibrium weakly decreases in ρ and weakly increases in m. Despite the fact that there are three possible cases in equilibrium, it will become obvious that case (iii), where the firms in the coalition do not participate in the contest, is not relevant. Such a case cannot be the outcome of a subgame-perfect equilibrium, because, in this case, the contingent fee is so high that all of the technology recipients do not have an incentive to compete if they join the coalition. Thus, our focus is on cases (i) and (ii), and the next remark is derived from the critical value of ρ that differentiates the two cases. While all firms in the coalition have access to the advanced technology in both cases, the firms not in the coalition still compete using their inferior technology in case (i) but leave the competition in case (ii). We formalize the outcome of case (ii) by deeming firm 1's technology "radical" because if it is adopted by all other firms in the coalition for a given license fee, the inferior firms not in the coalition will leave the competition. As a result, firm 1 can deter all outsiders from competing by sharing its superior technology only with the firms in the coalition.

Definition 1 *Given $m \in \{1, ..., n-2\}$, we say that firm 1's technology is radical with respect to m and ρ, if each firm $i \notin C_m$ does not participate in the competition whenever a coalition C_m is formed and the contingent rate of the coalition's license fee is ρ.*

The objective of this definition is to derive the minimum value of c for a given pair of m and ρ such that the firms outside the coalition will not participate in the contest. We show below that such a value of c is increasing in ρ and decreasing in m.

Remark 2 *Define*

$$\bar{c}(\rho|m) \equiv \frac{m+1}{m(1-\rho)}. \tag{10}$$

Firm 1's technology is radical with respect to m and ρ if and only if $c \geq \bar{c}(\rho|m)$.

(i) For a given m, $\bar{c}(\rho|m)$ is increasing in ρ.

(ii) For a given ρ, $\bar{c}(\rho|m)$ is decreasing in m.

Proof. This threshold value of c is derived from the condition $\rho \leq \frac{mc-m-1}{mc}$ stated in Proposition 1 (ii). The properties of $\bar{c}(\rho|m)$ follow immediately. ■

Next, we let $x_i^e(\rho|m)$ denote firm i's investment in equilibrium. We find that when only firms in the coalition are active in equilibrium, each firm in the coalition invests the same amount and therefore has the same probability of winning the contest.

Remark 3 *If $m = n-1$ or $c \geq \bar{c}(\rho|m)$, then*

$$x_i^e(\rho|m) = \frac{m(1-\rho)}{(m+1)^2} R \tag{11}$$

for all $i \in C_m$, and the probability of winning the contest in equilibrium is $\frac{1}{m+1}$ for all firms in C_m.

Proof. Given $m = n-1$, the expression for X^e in (7) is the same as X^e in (8), where $m < n-1$ and $c \geq \bar{c}(\rho|m)$. Given X^e in (8), firm 1's probability of winning, given by (38), and firm i's probability of winning for $i = 2, ..., m+1$, given by (39), are equal to $\frac{1}{m+1}$. ■

Example: $n = 3$

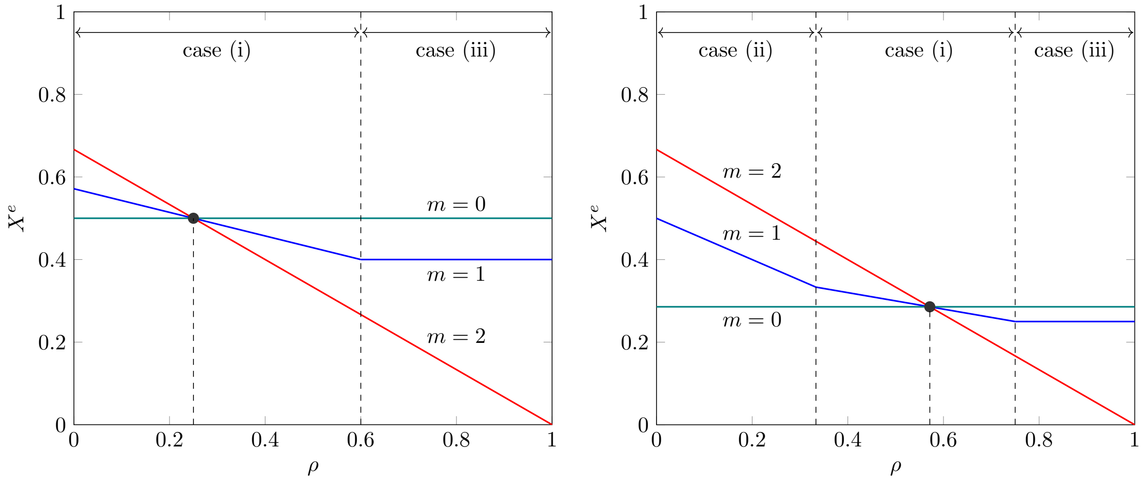
As an example, consider a contest with three firms. Based on Proposition 1, we obtain the following expression for the aggregate investment in equilibrium for given values of c, m, and ρ.

$$X^e(\rho|m) = \begin{cases} \left(\frac{2}{2c+1}\right)R & \text{if } m = 0 \\ \left(\frac{1-\rho}{2}\right)R & \text{if } m = 1 \text{ and } \rho \leq \frac{c-2}{c} \\ \left(\frac{2-\rho}{c+2}\right)R & \text{if } m = 1 \text{ and } \frac{c-2}{c} < \rho < \frac{c}{c+1} \\ \left(\frac{1}{c+1}\right)R & \text{if } m = 1 \text{ and } \rho \geq \frac{c}{c+1} \\ \left(\frac{2-2\rho}{3}\right)R & \text{if } m = 2 \end{cases} \tag{12}$$

We plot X^e on ρ for all possible values of m given $c = \frac{3}{2}$ and $c = 3$ in Figure 2. For $c = \frac{3}{2}$, case (ii) $\rho \leq \frac{c-2}{c}$ is not possible, case (i) and case (iii) arise when ρ is smaller and greater than $\frac{3}{5}$, respectively. In Figure 2a, X^e is increasing in m if and only if ρ is less than $\frac{1}{4}$. In Figure 2b, $c = 3$ and X^e is increasing in m if and only if ρ is less than $\frac{4}{7}$. Using X^e derived for the case $n = 3$, we can calculate firm i's effort level $x_i^e(\rho|m)$ and expected payoff $\pi_i^e(\phi, \rho|m)$ for $i = 1, 2, 3$ for each case as in Table 1. As suggested by Remark 3, $x_1^e(\rho|2) = x_2^e(\rho|2) = x_3^e(\rho|2)$ and $x_1^e(\rho|1) = x_2^e(\rho|1)$ in case (ii) so that each firm in the coalition has an equal probability of winning. We use the information in Table 1 to derive the license fees that are a bargaining solution in stage 2 as a numerical example in the following section.

5 Bargaining Solutions in Stage 2

In the previous section, we derive an equilibrium for a subgame appearing in stage 3, given the m chosen in stage 1 and licensing contract (ϕ, ρ) determined in stage 2. In this section, we derive solutions of the licensing contract for the given value of $m \in \{1, ...n-1\}$ with the two bargaining solution concepts: (1) all firms in the coalition adopt an NB solution and (2) firm 1 plays a TIOLI strategy.



(a) $c = \frac{3}{2}$ (b) $c = 3$

Figure 2: Total effort in equilibrium for $n = 3$

	$m=0$ (autarky)	$m=1$ case (i)	$m=1$ case (ii)	$m=1$ case (iii)	$m=2$
$x_1^e(\rho\|m)$	$\frac{2(2c-1)R}{(2c+1)^2}$	$\frac{[c(1-\rho)^2+\rho](2-\rho)R}{(c+2)^2(1-\rho)}$	$\frac{(1-\rho)R}{4}$	$\frac{cR}{(c+1)^2}$	$\frac{2(1-\rho)R}{9}$
$x_2^e(\rho\|m)$	$\frac{2R}{(2c+1)^2}$	$\frac{[c(1-\rho)-\rho](2-\rho)R}{(c+2)^2(1-\rho)}$	$\frac{(1-\rho)R}{4}$	0	$\frac{2(1-\rho)R}{9}$
$x_3^e(\rho\|m)$	$\frac{2R}{(2c+1)^2}$	$\frac{[2-c(1-\rho)](2-\rho)R}{(c+2)^2}$	0	$\frac{R}{(c+1)^2}$	$\frac{2(1-\rho)R}{9}$
$\pi_1^e(\phi,\rho\|m)$	$\frac{(2c-1)^2R}{(2c+1)^2}$	$\frac{(c+\rho)^2R}{(c+2)^2} + \phi R + \frac{[c(1-\rho)-\rho](2-\rho)\rho R}{(c+2)^2(1-\rho)}$	$\frac{(1+3\rho)R}{4} + \phi R$	$\frac{c^2R}{(c+1)^2}$	$\frac{(1+8\rho)R}{9} + 2\phi R$
$\pi_2^e(\phi,\rho\|m)$	$\frac{R}{(2c+1)^2}$	$\frac{[c(1-\rho)-\rho]^2R}{(c+2)^2(1-\rho)} - \phi R$	$\frac{(1-\rho)R}{4} - \phi R$	0	$\frac{(1-\rho)R}{9} - \phi R$
$\pi_3^e(\phi,\rho\|m)$	$\frac{R}{(2c+1)^2}$	$\frac{[2-c(1-\rho)]^2R}{(c+2)^2}$	0	$\frac{R}{(c+1)^2}$	$\frac{(1-\rho)R}{9} - \phi R$

Table 1: Equilibrium effort and payoff for each firm for $n = 3$

The NB solution solves the following problem:

$$\max_{\phi,\rho} W(\phi,\rho|m) \equiv \Pi_{i=1}^{m+1}[\pi_i^e(\phi,\rho|m) - \pi_i^a]$$
$$\text{s.t. } \pi_i^e(\phi,\rho|m) - \pi_i^a \geq 0 \text{ for all } i \in C_m,$$
$$0 \leq \phi \leq 1, \text{ and } 0 \leq \rho \leq 1, \tag{13}$$

where the condition $\pi_i^e(\phi,\rho|m) - \pi_i^a \geq 0$ is the participation constraint of each firm i in the coalition. On the other hand, firm 1's TIOLI solution solves the following problem:

$$\begin{aligned}
&\max_{\phi,\rho}\ \pi_1^e(\phi,\rho|m)\\
&\text{s.t. } \pi_i^e(\phi,\rho|m)-\pi_i^a\geq 0 \text{ for all } i\in C_m,\\
&\qquad 0\leq\phi\leq 1,\ \text{ and } 0\leq\rho\leq 1.
\end{aligned}\tag{14}$$

For a given value of $m \in \{1, ..., n-1\}$ in stage 1, we solve the two problems above while using the equilibrium outcomes drawn by Proposition 1. We let $\left(\hat{\phi}(m), \hat{\rho}(m)\right)$ and $\left(\tilde{\phi}(m), \tilde{\rho}(m)\right)$ denote the NB and TIOLI solutions, respectively, and analyze the two general cases, $m = n-1$ and $m \leq n-2$, separately below.

5.1 The case $m = n - 1$

This case implies that a grand coalition forms in stage 1, i.e., the network structure chosen by firm 1 is complete bipartite. Given $m = n - 1$, the equality participation constraint of each firm (defined in detail below) is linear in ϕ and ρ so that the NB and TIOLI solutions can be obtained explicitly and intuitively. We obtain the following results.

Proposition 2 *Suppose that a complete bipartite network is formed in stage 1.*

1. *The Nash bargaining solution in the subgame starting in stage 2, given $m = n-1$, is*

$$\hat{\phi}(m) = 0 \textit{ and } \hat{\rho}(m) = \frac{(n-1)(c-1)[(n-1)(n+1)(c-1)+4n]}{(n+1)[(n-1)c+1]^2}. \tag{15}$$

2. *The TIOLI solution in the subgame starting in stage 2, given $m = n-1$, is*

$$\tilde{\phi}(m) = 0 \textit{ and } \tilde{\rho}(m) = \frac{(n-1)(c-1)[(n-1)(c+1)+2]}{[(n-1)c+1]^2}. \tag{16}$$

Proof. See Appendix A.2.

If firm 1 shares its advanced technology with all other firms, i.e., $m = n-1$, the fixed fee component of the license will be zero regardless of whether it is the NB or TIOLI solution, and the NB contingent rate will be strictly smaller than the TIOLI contingent rate. The fact that $\tilde{\rho}(n-1) > \hat{\rho}(n-1)$ can be intuitively explained by the relative bargaining power of firm 1 between TIOLI and NB. The reason why $\tilde{\phi}(n-1) = \hat{\phi}(n-1) = 0$ is as follows.

There are two possible outcomes when $m = n-1$, i.e., either firm 1 or one of the licensees is the winner of the R&D competition. As ρ increases, firm 1's expected payoff increases because of the increase in the contingent fee transferred from one of the licensees when firm 1 does not win the contest. As ϕ increases, firm 1's expected payoff increases regardless of who wins the contest. Since firm 1's marginal effects from ρ and ϕ are $\frac{(n+1)(n-1)}{n^2}R$ and $(n-1)R$, respectively, the marginal rate of substitution between ϕ and ρ is $\frac{n^2}{n+1}$. For a licensee, as ρ increases, its expected payoff decreases because of the increase in the contingent fee transferred to firm 1 in case it wins the contest. However, an increase of ϕ decreases each licensee's expected payoff regardless of the outcome of the contest. The marginal effects with respect to ρ and ϕ for each licensee are $-\frac{1}{n^2}R$ and $-R$, respectively, so the marginal rate of substitution between ϕ and ρ is n^2. Since each licensee is willing to accept a larger increase in ρ for a fixed reduction in ϕ than its counterpart, all parties can improve their own expected payoff by substituting ϕ with ρ at a rate between the two marginal rates of substitution. As a result, the bargaining procedure entices them to choose $\phi = 0$.

Example: $n = 3$

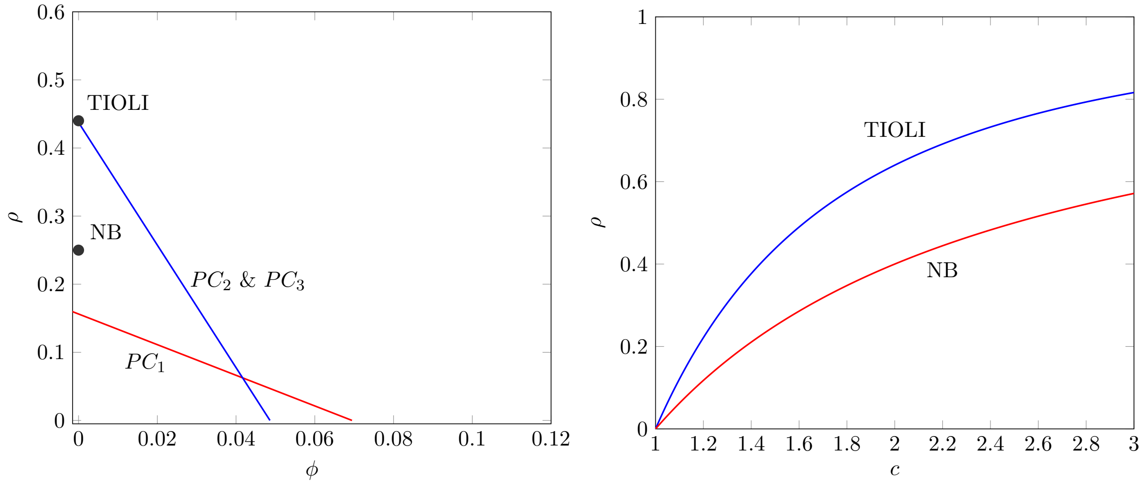
Given $\pi_i^e(\phi, \rho|2)$ and π_i^a, for $i = 1, 2, 3$, derived for the case $n = 3$, we define firm i's *equality participation constraint* by $PC_i \equiv \{(\phi, \rho)|\pi_i^e(\phi, \rho|m) - \pi_i^a = 0\}$. Based on Table 1, we have

$$PC_1 : \rho = \frac{(4c-1)(c-1)}{(2c+1)^2} - \frac{9}{4}\phi \tag{17}$$

and

$$PC_2 \ \& \ PC_3 : \rho = \frac{4(c+2)(c-1)}{(2c+1)^2} - 9\phi. \tag{18}$$

Thus, the relationship between ϕ and ρ for each PC_i is linear. If we plot each PC_i on the (ϕ, ρ) Cartesian plane, we will find that PC_1 is flatter than PC_2 (which is the same



(a) Bargaining solutions given $c = \frac{3}{2}$

(b) Contingent fees given any $c > 1$

Figure 3: NB and TIOLI licensing fees given $n = 3$ and $m = 2$

as PC_3) and that the intersection of PC_1 and PC_2 lies strictly in the first quadrant. As a result, the feasible set of (ϕ, ρ) according to (13) is nonempty. For example, we plot PC_i for $i = 1, 2, 3$ given $c = \frac{3}{2}$ in Figure 3a. The feasible set is the triangular region above PC_1 and below PC_2.

Given the feasible region, we seek to find a couple (ϕ, ρ) maximizing $W(\phi, \rho|2)$. First, we claim that any interior point within the feasible region cannot be a solution and, therefore, $\hat{\phi}(2) = 0$. The reason is as follows. Pick an arbitrary interior point of (ϕ', ρ') from the feasible region in Figure 3a. We can then draw the isopayoff line of firm 1 that passes through (ϕ', ρ'), i.e., $\left\{(\phi, \rho)|\pi_1^e(\phi, \rho|2) - \pi_1^e(\phi', \rho'|2) = 0\right\}$ and the isopayoff line of firm 2 that passes through (ϕ', ρ'), i.e., $\left\{(\phi, \rho)|\pi_2^e(\phi, \rho|2) - \pi_2^e(\phi', \rho'|2) = 0\right\}$. It follows that there exists another interior point (ϕ'', ρ''), where $\phi'' < \phi'$ and $\rho'' > \rho'$, between the two isopayoff lines that makes both firms strictly better off and hence strictly increases $W(\phi, \rho|2)$. Therefore, the NB solution must be on the vertical axis. By substituting $\phi = 0$ into $W(\phi, \rho|2)$, it follows that

$$\hat{\phi}(2) = 0 \text{ and } \hat{\rho}(2) = \frac{2(c-1)}{2c+1}. \tag{19}$$

Given $c = \frac{3}{2}$, we find that $\hat{\rho}(2) = \frac{1}{4}$ as depicted by point NB in Figure 3a.

Now consider the TIOLI solution. As we see in the NB regime, PC_2 and PC_3 are identical as given by (18), and illustrated in Figure 3a for $c = \frac{3}{2}$. We find that firm 1's payoff-maximizing (ϕ, ρ) lies at the top corner of the feasible region. This corner solution can be understood intuitively. Note that firm 1's payoff $\pi_1^e(\phi, \rho|2)$ linearly increases with both ϕ and ρ. Thus, firm 1 would set ϕ and ρ as great as possible while guaranteeing the other firms just the minimum level of payoff as much as they reluctantly accept the offer, i.e., autarky payoff π_2^a. In other words, firm 1 would choose a (ϕ, ρ) on PC_2. Since PC_2 is steeper than the isopayoff line of firm 1, firm 1 would set the value of ϕ as low as possible and instead increase the value of ρ to the maximum along PC_2. It follows that

$$\tilde{\phi}(2) = 0 \text{ and } \tilde{\rho}(2) = \frac{4(c+2)(c-1)}{(2c+1)^2}. \tag{20}$$

Given $c = \frac{3}{2}$, we find that $\tilde{\rho}(2) = \frac{7}{16}$ as depicted by point TIOLI in Figure 3a.

We plot the contingent fees according to the NB and TIOLI solutions for all values of $c > 1$ in Figure 3b. Note that the fixed fee is always zero under both NB and TIOLI regimes and the contingent fees is always lower under NB than under TIOLI.

5.2 The case $m \leq n - 2$

In this case, the network is incomplete bipartite. Since there is at least one firm that is not in the coalition, we have to take into account whether these firms will be active or not in equilibrium for a given ρ to be chosen in stage 2. We find that the NB and TIOLI solutions depend on the ranges of both c and m. If at least half of the low-tech firms are in the coalition with firm 1, $i.e., m \geq \frac{n-1}{2}$, then the critical value of c (making the firms that do not belong to the coalition active or inactive in equilibrium) becomes $\frac{n-m}{n-m-1}$. If less than half of the low-tech firms are in the coalition with firm 1, $i.e., m < \frac{n-1}{2}$, then that critical value of c becomes $\frac{m+1}{m}$.[5] When c is lower than the threshold of each case, all firms are active and we can derive the NB and TIOLI solutions in the same way we did when $m = n-1$, except that the equality participation constraints of the firms are nonlinear when

[5] The critical values of c for given $m = 1$ and $m = n - 2$ are equal, those for given $m = 2$ and $m = n - 3$ are equal, and so on.

$m \leq n-2$. While the results for this case are conceptually similar to the case $m = n-1$, the expressions of the bargaining solutions are not compact and, for some given values of c and m, are not explicitly presentable. Therefore, we only include the results for the given c larger than the threshold in the following proposition, and present the rest of the results, i.e., when c is smaller than the threshold, in the proof of the proposition.

Proposition 3 *Suppose that an incomplete bipartite network is formed in stage 1 and* $c \geq \check{c}(m,n)$ *where*

$$\check{c}(m,n) \equiv \max\left\{\frac{m+1}{m}, \frac{n-m}{n-m-1}\right\}. \tag{21}$$

1. *The Nash Bargaining solution in a subgame starting in stage 2, given* $m \leq n-2$, *is*

$$\hat{\phi}(m) = \frac{[(n-1)c-n+2]^2-1}{[(n-1)c+1]^2(m+1)} - \frac{\hat{\rho}(m)}{m+1} \tag{22}$$

and

$$\hat{\rho}(m) = 1 - \frac{m+1}{mc}. \tag{23}$$

2. *The TIOLI solution in a subgame starting in stage 2, given* $m \leq n-2$, *is*

$$\tilde{\phi}(m) = \frac{[(c-c\tilde{\rho}(m)-1)(n-m-1)+1-\tilde{\rho}(m)]^2}{[(n-m-1)c+m+1]^2(1-\tilde{\rho}(m))} - \frac{1}{[(n-1)c+1]^2} \tag{24}$$

and

$$\tilde{\rho}(m) = 1 - \frac{m+1}{mc}. \tag{25}$$

Proof. See Appendix A.3.

Contrasting the bargaining solutions presented in Proposition 2 where $\tilde{\phi}(m) = \hat{\phi}(m) = 0$ and $\tilde{\rho}(m) > \hat{\rho}(m) > 0$, Proposition 3 suggests that, given $m \leq n-2$ and $c \geq \check{c}(m,n)$, then $\tilde{\phi}(m) > \hat{\phi}(m) > 0$ and $\tilde{\rho}(m) = \hat{\rho}(m) \geq 0$. Note that by substituting ρ with $\hat{\rho}(m)$ given by (23) or $\tilde{\rho}(m)$ given by (25) into (10), we have $\bar{c}(\rho|m) = c$. Thus, the value of ρ in the licensing fee solutions (regardless of NB or TIOLI) will be the maximum value that keeps firm 1's technology radical with respect to any given $c \geq \check{c}(m,n)$ and any $m \in \{1, ..., n-2\}$ chosen in stage 1. In other words, the contingent fee derived in Proposition 3 is the highest value of ρ that can deter all the firms outside of the coalition from competing in the contest. Since the contingent rates for the two bargaining concepts are identical and firm 1 has more bargaining power under TIOLI, it is intuitive to find that $\tilde{\phi}(m) > \hat{\phi}(m)$.

Example: $n = 3$

Given $n = 3$, we find that $\check{c}(1, 3) = 2$. Given $c \geq 2$, then only firms 1 and 2 participate. Given $m = 1$ case (ii) of Table 1, PC_1 and PC_2 are represented by

$$PC_1 : \rho = \frac{(6c-1)(2c-3)}{3(2c+1)^2} - \frac{4}{3}\phi \tag{26}$$

and

$$PC_2 : \rho = \frac{(2c+3)(2c-1)}{(2c+1)^2} - 4\phi \tag{27}$$

where $\rho \leq \frac{c-2}{c}$. Thus, we have a linear and negative relationship between ϕ and ρ for both PC_1 and PC_2. Following the approach for the case $m = 2$ discussed above, we find that an interior point between PC_1 and PC_2 cannot be a solution. In fact, we find that the NB solution is such that $\rho = \frac{c-2}{c}$ and the optimal value of ϕ that maximizes $W(\phi, \rho|1)$ given $\rho = \frac{c-2}{c}$. It follows that

$$\hat{\phi}(1) = \frac{7c+2}{2c(2c+1)^2} \text{ and } \hat{\rho}(1) = \frac{c-2}{c}. \tag{28}$$

We show the feasible region for $c = 3$ in Figure 4. The region above the dashed line corresponds to contest equilibria where all three firms compete as in case (i). The region on and below the dashed line corresponds to contest equilibria where only firm 1 and firm 2 compete as in case (ii). The NB solution is $\left(\frac{23}{294}, \frac{1}{3}\right)$ as depicted by point NB in Figure 4

Next, we analyze the TIOLI solution. Given $c \geq 2$, firm 3 will not participate in the contest. Firm 1 will choose (ϕ, ρ) on PC_2, i.e., $\phi(\rho) = \frac{(1-\rho)}{4} - \frac{1}{(2c+1)^2}$, that maximizes firm 1's expected payoff. It follows that

$$\tilde{\phi}(1) = \frac{4c^2+2c+1}{2c(2c+1)^2} \text{ and } \tilde{\rho}(1) = \frac{c-2}{c}. \tag{29}$$

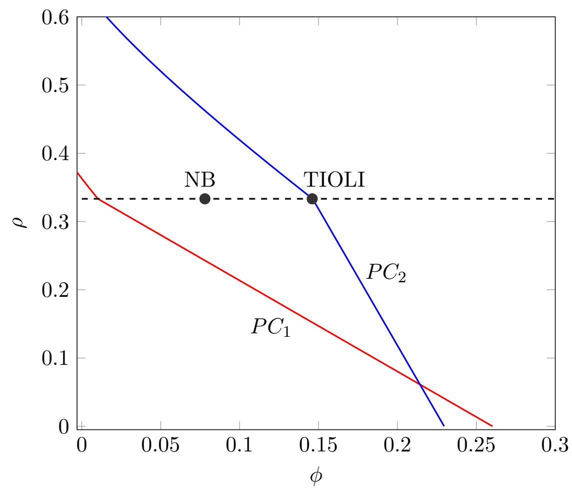


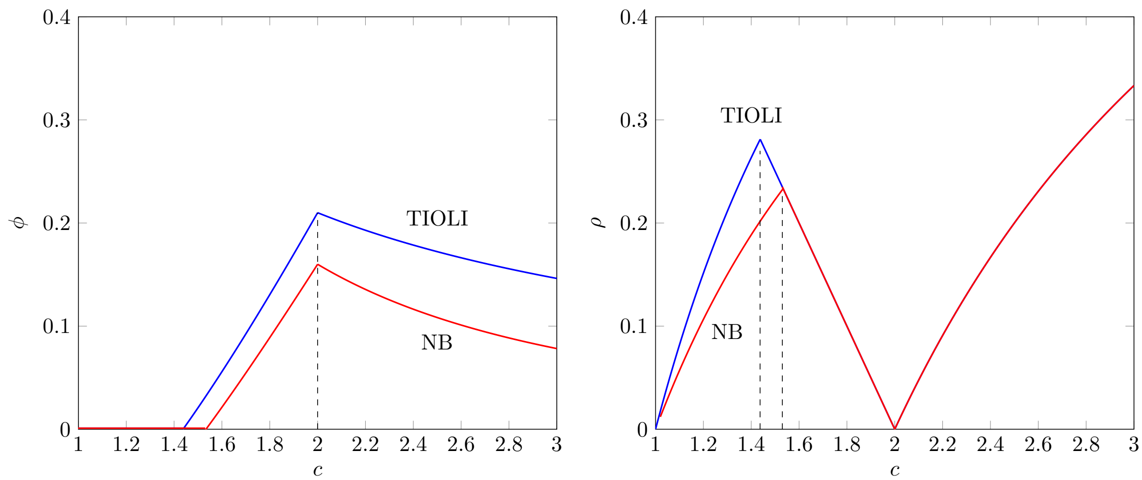
Figure 4: NB and TIOLI licensing fees given $n = 3$ and $m = 1$ when $c = 3$. If $c > 2$, then there exists a threshold value of $\rho > 0$, depicted by the dashed line, such that firm 3 will stay in (or leave) the competition whenever the license fee combination is above (or on/below) the dashed line.

Given $c = 3$, we find that $\tilde{\phi} = \frac{43}{294}$ and $\tilde{\rho}(1) = \frac{1}{3}$ as depicted by point TIOLI in Figure 4.

For the sake of completeness, we plot $\hat{\phi}(1)$, $\tilde{\phi}(1)$, $\hat{\rho}(1)$, and $\tilde{\rho}(1)$ on c for all $c > 1$ in Figure 5. See the derivation of $\tilde{\phi}(1)$, $\hat{\rho}(1)$, and $\tilde{\rho}(1)$ given $c \in (1, 2)$ in Appendix A.3. We find that $\hat{\phi}(1) \leq \tilde{\phi}(1)$ and $\hat{\rho}(1) \leq \tilde{\rho}(1)$ for all c. Specifically, $\hat{\rho}(1) < \tilde{\rho}(1)$ whenever $\hat{\phi}(1) = \tilde{\phi}(1)$, and $\hat{\phi}(1) < \tilde{\phi}(1)$ whenever $\hat{\rho}(1) = \tilde{\rho}(1)$.

6 Coalition in Stage 1

In this section, we use the bargaining solutions obtained in stage 2 to derive an optimal network structure represented by the value of m chosen by firm 1 in stage 1. In the previous section, we present the bargaining solutions given $m = n-1$ and $c > 1$ in Proposition 2, and given $m \leq n - 2$ and $c \geq \check{c}(m, n)$ in Proposition 3. As we show in the proof of Proposition 3, for a given $m \leq n - 2$, there is a range c (near 1) such that the bargaining solution cannot be expressed explicitly. Therefore, to derive an optimal value of m for any n and any $c > 1$, a numerical approach is necessary. In this section, our focus is on the case $c \geq 2$, not only because the results can be explicitly derived, but also because it has an



(a) Fixed fees given $c > 1$

(b) Contingent fees given $c > 1$

Figure 5: NB and TIOLI licensing fees given $n = 3$ and $m = 1$

implication on entry deterrence. In particular, we find that $c \geq 2$ is a necessary condition for entry deterrence to occur in equilibrium given any $n \geq 3$.

For the sake of completeness, we demonstrate how to derive an optimal network given any $c > 1$ for the cases $n = 3$ and $n = 4$. We conclude this section by examining whether the derived network structures are stable by allowing firms in the coalition to drop out after learning the license fee at the end of stage 2.

6.1 Equilibrium network

Let $\hat{m}(n)$ and $\tilde{m}(n)$ be firm 1's optimal choices of m given n in the subgame perfect equilibrium under NB and TIOLI regimes, respectively. First, we present the result given $c \geq 2$. Then, we discuss the case given $c \in (1, 2)$ and $n = 3, 4$.

Proposition 4 *Suppose $c \geq 2$.*

1. *Under NB regime,*

$$\hat{m}(n) = \begin{cases} n - 1 & \text{if } n = 3, 4 \\ 1 & \text{if } n \geq 5. \end{cases} \tag{30}$$

2. *Under TIOLI regime, $\tilde{m}(n) = n - 1$ for all $n \geq 3$.*

Proof. See Appendix A.4.

Firm 1 is facing the licensing dilemma in stage 1 of the game. As it increases the number of licensees, income from licensing rises while its competitive advantage diminishes. Given $c \geq 2 \geq \check{c}(m,n)$, Remarks 2 and 3 suggest that firms outside the coalition will not participate in the contest and each firm in the coalition will have an equal probability of winning the competition. Thus, given $m \in \{1, ...n-1\}$, firm 1's probability of winning is $\frac{1}{m+1}$.

Under NB regime, each firm in the coalition has equal bargaining power. As the coalition expands, we find that firm 1's expected payoff decreases as m increases from 1 to $n-2$. Since $m = 2, ..., n-2$ are strictly dominated, there are only two candidates left for the optimal value of m, i.e., 1 and $n-1$. For $m = 1$, firm 1 can collect a license fee from its one and only licensee while deter all other firms from competing. For $m = n-1$, firm 1 can collect license fees from $n-1$ licensees while no firm is deterred from the competition. We find from Proposition 4 that the entry-deterring effect dominates the revenue effect if and only if $n \geq 5$. That is, when $n \geq 5$, firm 1 will form a coalition with only one other firm and deter $n-2$ firms from entering the contest. As a result, the probability of winning for firm 1 becomes $\frac{1}{2}$. When $n = 3$ or 4, firm 1 will form a coalition with all other firms and every firm will remain in the competition with an equal probability of winning (i.e., $\frac{1}{3}$ for $n = 3$ and $\frac{1}{4}$ for $n = 4$).

Under TIOLI regime, firm 1 faces a similar dilemma. However, because it is the only firm that has bargaining power, it can keep increasing ϕ and ρ as long as the expected payoff of its licensees is not lower than in autarky. Proposition 4 suggests that the revenue effect dominates the entry-deterring effect for all $n \geq 3$, so firm 1 will choose $m = n-1$ and form a grand coalition in equilibrium.

Based on the above results regarding firm 1's radical technology, we find that from the social welfare aspect, the outcome is a Pareto improvement under the TIOLI regime. Even though firm 1 takes advantage of the TIOLI offer, no firm is made worse off. In contrast, under NB, firm 1's relative bargaining power declines as the coalition expands. As a result, the marginal revenue from an increase in m is offset by the reduction in the probability of winning. When the number of firms is five or more, firm 1 will have only one licensee and deter all other firms from participating. While firm 1 and the licensee are better off, all other firms are made worse off. At a glance, equalizing all the firms' status in the bargaining within the coalition may seem desirable for all technology recipients. However, if the advanced firm can determine the size of the coalition, it may form a technology cartel that leads to a duopoly outcome in the competition. An intervention by a policymaker or the contest organizer may be necessary to facilitate technology diffusion or to level the playing field. For example, if the advanced firm is sufficiently compensated, it will be willing to share its technology with all other firms and, as a result, every firm has an equal chance of winning in equilibrium.

Finally, we focus on the case of a small contest where $n \in \{3, 4\}$ with $c \in (1, 2)$. We compare firm 1's expected payoffs given all possible values of m, i.e., $m = 0, 1, 2$ for $n = 3$ and $m = 0, 1, 2, 3$ for $n = 4$. Given c close to 1, we can not explicitly derive the solution for the license fee or firm 1's expected payoff so we need to rely on a numerical analysis. We find that regardless of the concepts determining the licensing fee, NB or TIOLI solution, firm 1 will form a complete bipartite network in stage 1.

Proposition 5 *Given $c \in (1, 2)$ and $n \in \{3, 4\}$, then $\hat{m}(n) = \tilde{m}(n) = n - 1$.*

Proof. See Appendix A.5.

In a small contest, under either NB or TIOLI regime, it is best for firm 1 to keep all other firms in the coalition for any $c > 1$. The benefit from leaving one or two firms out of the coalition (and letting them compete with the inferior technology or stay out of the competition) does not outweigh the income loss of the license fee that firm 1 could have collected from those firms, even under the NB regime.

6.2 Network stability

Up to this point, we assumed that firms join the coalition whenever their expected payoff after joining is greater than their autarky payoff and that the coalition breaks down whenever one of the firms in the coalition rejects the proposed license fee. Now, instead of returning to autarky, we assume that if a firm in the coalition rejects the license fee, it leaves the coalition, and the other firms remaining in the coalition renegotiate the license contract. In this case, the game will proceed as illustrated in Figure 6.

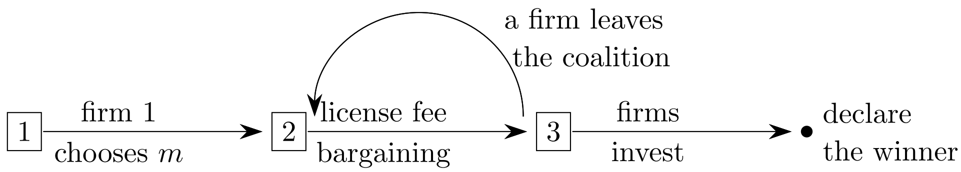


Figure 6: The timeline of the game when a firm is allowed to leave the coalition after learning the license fee

We formally define stability of the coalition as follows.

Definition 2 *When a coalition C_m is formed and the licensing fee structure $(\phi(m), \rho(m))$ is determined in stage 2, we say that the coalition is pairwise stable if*

$$\pi^e_{m+1}(\phi(m), \rho(m)|m) \geq \pi^e_{m+1}(\phi(m-1), \rho(m-1)|m-1). \tag{31}$$

The above definition coincides with the concept of pairwise stability in the economics of network. We use firm $m+1$ as the representative firm for those who receive technology from firm 1. We omit firm 1's payoff conditions in the definition because firm 1 is the firm that forms the coalition by choosing m in stage 1.

Proposition 6 *Consider the coalition in equilibrium derived above.*

1. *Suppose $c \geq 2$. The network structure in equilibrium under NB and TIOLI regimes derived in Proposition 4 is pairwise stable.*

2. *Suppose $c \in (1, 2)$ and $n = 3, 4$. The network structure in equilibrium under NB regime derived in Proposition 5 is pairwise stable. The network structure in equilibrium under TIOLI regime derived in Proposition 5 is pairwise stable if and only if $c \geq \frac{2n-3}{n-1}$.*

Proof. See Appendix A.6.

The first result of Proposition 6 is intuitive. When firm 1's technology is radical, leaving the coalition means not participating in the competition and receiving no payoff at all. The consequence of leaving the coalition is worse than being in autarky, so any coalition formed when $c \geq 2$ will be stable. In contrast, when $n \leq 4$ and c is small enough, it is possible for the coalition to be unstable under TIOLI. Since firm 1's technology is not radical, firms that left the coalition can still compete using their inferior technology. After firm 1 has revised the license fee by making a TIOLI offer to the firms that remain in the coalition, the high contingent fee cannot deter the firms that left the coalition from competing and, as a result, it is possible for those firms to be made better off than in autarky. We find that this unstable outcome can occur under TIOLI given $n \in \{3, 4\}$ when $c < \frac{2n-3}{n-1}$.

7 Conclusion

In this paper, we theoretically model an R&D competition in which one of the firms possesses an advanced technology that can be licensed out to its competitors. In return, each of the technology recipients pays the technology leader a two-part tariff consisting of fixed and contingent components. We use Tullock's contest as the underlying mechanism in determining the R&D winner while allowing for an endogenous licensing coalition and fee structure. We find that a coalition always exists in equilibrium, while the coalition network structure depends on the total number of firms, the bargaining power of licensor, and the significance of the technology.

This is the first paper that analyzes technology licensing structure and fee in the canonical R&D contest model. We allow for endogenous licensing structure and fee determination. We relate each firm's bargaining power and the significance of the licensed technology to the network structure and the license fee in equilibrium. Using the Nash bargaining concept, we find that the advanced technology is shared with all the firms if the number of competing firms is less than five and shared with only one other firm if the number of competing firms is at least five and the technology is radical. If the advanced firm plays a take-it-or-leave-it strategy, it will always form a grand coalition and share its technology with all other firms. Thus given a radical technology and the number of firms of five or more, the equilibrium outcome under the take-it-or-leave-it regime is a Pareto improvement, while the equilibrium outcome under the Nash bargaining regime is not.

Our theory implies that when the license fee structure does not provide the advanced firm with sufficient compensation, the firm may strategically form a small technology cartel to deter other firms not in the cartel from competing. Our theory also suggests that by sharing its technology with all other firms, there would be no advantage in the competition (i.e., each firm would have an equal probability of winning), while the advanced firm would receive a higher expected payoff than do the technology recipients. A key assumption of our model is that the winner-selection process is described by an imperfectly discriminating contest success function. That is why the advanced firm has an incentive to share its technology with rival firms through the license contract. So, this nature of competition, i.e., the strongest firm is not always the winner in R&D contest, can be thought of as one of the key motivations for open innovation. In such a situation, players strategically share their skills and knowledge, and consequently reach a mutually beneficial point. This series of interactions between the players is being implemented in the process of open innovation.

Since this is the first theoretical study of licensing R&D technology in Tullock's contest, there are many directions to extend our framework for future research. One might investigate a contest where the contest organizer can choose the license fee and use it to manipulate the outcome of the contest. Due to the uncertainty in the R&D competition, one might also examine the effects of risk attitudes on a firm's decision to license out or license in.

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A Proofs

A.1 Proof of Proposition 1

The first-order conditions for an interior solution of all the firms (i.e., all firms participate in the contest) are given by

$$\frac{X - x_1}{X^2}R - \sum_{i=2}^{m+1} \frac{x_i}{X^2}\rho R - 1 = 0, \tag{32}$$

$$\frac{X - x_i}{X^2}(1-\rho)R - 1 = 0, \text{ for } i = 2, ..., m+1 \tag{33}$$

$$\frac{X - x_i}{X^2}R - c = 0, \text{ for } i = m+2, ..., n. \tag{34}$$

which imply the following share functions

$$s_1(X) = 1 - \rho \sum_{i=2}^{m+1} s_i(X) - \frac{X}{R}, \tag{35}$$

$$s_i(X) = 1 - \frac{X}{(1-\rho)R}, \text{ for } i = 2, ..., m+1 \tag{36}$$

$$s_i(X) = 1 - \frac{cX}{R}, \text{ for } i = m+2, ..., n \tag{37}$$

whenever $s_i(X) > 0$. Note that $s_1(X)$ is conditional on $s_i(X)$ for $i = 2, ..., m+1$. It follows that

$$s_1(X) = \begin{cases} 1 - m\rho - \left[1 - \frac{m\rho}{1-\rho}\right]\frac{X}{R} & \text{if } X < (1-\rho)R \\ 1 - \frac{X}{R} & \text{if } (1-\rho)R \leq X < R \\ 0 & \text{if } X \geq R, \end{cases} \tag{38}$$

$$s_i(X) = \begin{cases} 1 - \frac{X}{(1-\rho)R} & \text{if } X < (1-\rho)R \\ 0 & \text{if } X \geq (1-\rho)R, \end{cases} \tag{39}$$

for $i = 2, ..., m+1$, and

$$s_i(X) = \begin{cases} 1 - \frac{cX}{R} & \text{if } X < \frac{R}{c} \\ 0 & \text{if } X \geq \frac{R}{c}, \end{cases} \tag{40}$$

for $i = m+2, ..., n$. Let us denote firm i's participation threshold by κ_i. According to $s_i(X)$ derived above, $\kappa_1 = R$, $\kappa_i = (1-\rho)R$ for $i = 2, ..., m+1$, and $\kappa_i = \frac{R}{c}$ for $i = m+2, ..., n$. Given $\rho \in [0,1]$ and $c > 1$, we have three possible cases.

Case 1. $\kappa_1 \geq \kappa_2 = ... = \kappa_{m+1} > \kappa_{m+2} = ... = \kappa_n$. It follows that

$$S(X) := \sum_{i=1}^{n} s_i(X) = \begin{cases} n - m\rho - \frac{[m+1+(n-m-1)c]X}{R} & \text{if } X < \frac{R}{c} \\ m + 1 - m\rho - \frac{(m+1)X}{R} & \text{if } \frac{R}{c} \leq X < (1-\rho)R \\ 1 - \frac{X}{R} & \text{if } (1-\rho)R \leq X < R \\ 0 & \text{if } X \geq R. \end{cases} \tag{41}$$

Since $S(X)$ is continuous given $X \geq 0$, $S(X) > 1$ given $X = 0$ and $S = 0$ given $X \geq R$, and $S(X)$ is strictly decreasing given $X \in (0, R)$, there exists a unique equilibrium where $S(X) = 1$ if $S(X)$ is strictly decreasing in X for all $X \in (0, R)$. There are two possible subcases.

Subcase 1.1. All firms participate in equilibrium. Setting $S(X) = 1$ yields X^e as in Case (i) of Proposition 1.

Subcase 1.2. Only firms $1, ..., m$ participate in the contest. If firms $m+2, ..., n$, do not participate, $s_i(X) = 0$ for $i = m+2, ..., n$, and the condition $S(X) = 1$ implies X^e as in Case (ii) of Proposition 1. We find that $X^e \geq \frac{R}{C}$ if and only if $\rho \leq \frac{mc-m-1}{mc}$.

Case 2. $\kappa_1 > \kappa_{m+2} = ... = \kappa_n > \kappa_2 = ... = \kappa_{m+1}$. It follows that

$$S(X) := \sum_{i=1}^{n} s_i(X) = \begin{cases} n - m\rho - \frac{[m+1+(n-m-1)c]X}{R} & \text{if } X < (1-\rho)R \\ n - m - \frac{[1+(n-m-1)c]X}{R} & \text{if } (1-\rho)R \leq X < \frac{R}{c} \\ 1 - \frac{X}{R} & \text{if } \frac{R}{c} \leq X < R \\ 0 & \text{if } X \geq R. \end{cases} \tag{42}$$

Since $S(X)$ is continuous given $X \geq 0$, $S(X) > 1$ given $X = 0$ and $S = 0$ given $X \geq R$, and $S(X)$ is strictly decreasing given $X \in (0, R)$, there exists a unique equilibrium where $S(X) = 1$ if $S(X)$ is strictly decreasing in X for all $X \in (0, R)$. There are two possible subcases.

Subcase 2.1. All firms participate in equilibrium. Setting $S(X) = 1$ yields X^e as in Case (i) of Proposition 1.

Subcase 2.2. Only firms $1, m+2, ..., n$ participate in the contest. If firms $2, ..., m+1$, do not participate, $s_i(X) = 0$ for $i = 2, ..., m+1$, and the condition $S(X) = 1$ implies X^e as in Case (iii) of Proposition 1. We find that $X^e \geq (1-\rho)R$ if and only if $\rho \geq \frac{(n-m-1)(c-1)+1}{(n-m-1)c+1}$.

Case 3. $\kappa_1 > \kappa_2 = ... = \kappa_n$. It follows that all firms participate in equilibrium and setting $S(X) = 1$ yields X^e as in Case (i) of Proposition 1.

A.2 Proof of Proposition 2

In autarky, each firm's expected payoff in equilibrium can be written as

$$\pi_1^a = \left[1 - \frac{(n-1)}{(n-1)c+1}\right]^2 R, \tag{43}$$

$$\pi_i^a = \left[1 - \frac{(n-1)c}{(n-1)c+1}\right]^2 R, \tag{44}$$

where $i = 2, ..., n$. Given that a coalition with $m = n-1$ is formed, each firm's expected payoff in equilibrium can be written as

$$\pi_1^e(\phi, \rho|m) = \left\{ \left(1 - \frac{X^e}{R}\right)^2 + m\rho \left[1 - \frac{X^e}{(1-\rho)R}\right] \frac{X^e}{R} + m\phi \right\} R \tag{45}$$

$$\pi_i^e(\phi, \rho|m) = \left\{ \left[1 - \frac{X^e}{(1-\rho)R}\right]^2 (1-\rho) - \phi \right\} R \tag{46}$$

where $i = 2, ..., n$, and X^e is given by (7).

NB solution

Given $m = n-1$, the NB solution solves the following problem.

$$\max_{\phi,\rho} W(\phi, \rho|n-1) = [\pi_1^e(\phi, \rho|n-1) - \pi_1^a][\pi_i^e(\phi, \rho|n-1) - \pi_i^a]^{n-1}$$
$$\text{s.t. } \pi_1^e(\phi, \rho|n-1) - \pi_1^a \geq 0, \text{ and } \pi_i^e(\phi, \rho|n-1) - \pi_i^a \geq 0 \; for \; i = 2, ..., n, \tag{47}$$

with $0 \leq \phi \leq 1$ and $0 \leq \rho \leq 1$. Define $PC_1 \equiv \{(\phi, \rho)|\pi_1^e(\phi, \rho|n-1) - \pi_1^a = 0\}$ and $PC_i \equiv \{(\phi, \rho)|\pi_i^e(\phi, \rho|n-1) - \pi_i^a = 0\}$ for $i = 2, ..., n$. Then, PC_1 and PC_i can be represented on the (ϕ, ρ) plane as

$$PC_1 : \rho = \frac{(n-1)(c-1)[c(n+1)(n-1) - n^2 + 2n + 1]}{(n+1)[c(n-1)+1]^2} - \frac{n^2}{n+1}\phi, \tag{48}$$

$$PC_i : \rho = \frac{(n-1)(c-1)[c(n-1) + n + 1]}{[c(n-1)+1]^2} - n^2\phi. \tag{49}$$

We find that the intersection of PC_1 and PC_i uniquely exists in the first quadrant of the (ϕ, ρ) plane. It follows that the feasible set of (ϕ, ρ) for the problem in (47) is a triangular region of the (ϕ, ρ) plane bordered by PC_1, PC_i, and the vertical axis ($\phi = 0$). Since PC_1 is flatter than PC_i as shown in (48) and (49), we find that any (ϕ, ρ) in the feasible region with $\phi > 0$ does not maximize W because W increases as ϕ decreases and ρ increases at a substitution rate between $\frac{n^2}{n+1}$ and n^2. Therefore, $\hat{\phi}(n-1) = 0$. By substituting $\phi = 0$ into $W(\phi, \rho|n-1)$ and maximizing it by choosing ρ, we obtain $\hat{\rho}(n-1)$ as in (15).

TIOLI solution

Since $\pi_1^e(\phi, \rho|n-1)$ increases in ϕ and ρ, firm 1 will choose (ϕ, ρ) on PC_i represented by (49) that maximizes $\pi_1^e(\phi, \rho|n-1)$. Firm 1's marginal rate of substitution for a 1 unit decrease in ϕ is n^2 units increase in ρ which is larger than $\frac{n^2}{n+1}$ that is needed by firm i to stay on PC_i. Therefore, firm 1 chooses $\tilde{\phi}(n-1) = 0$ and (49) implies that $\tilde{\rho}(n-1)$ is given by (16).

A.3 Proof of Proposition 3

Proposition 3 presents special cases (i-c and ii-b) of Proposition 7 (NB solutions) and Proposition 8 (TIOLI solutions) below.

Proposition 7 *Given $m \in \{1, ..., n-2\}$ determined in stage 1, the Nash Bargaining solution uniquely exists in each of the following cases in stage 2.*

(i) If $\frac{n-1}{2} \leq m < n-1$, we have:

(a) *For* $c \leq \tau$,

$$\hat{\phi}(m) = 0 \text{ and } \hat{\rho}(m) = \arg\max_{\rho} W(0, \rho|m).$$

(b) *For* $\tau < c < \frac{n-m}{n-m-1}$,

$$\hat{\phi}(m) = \frac{1}{m+1}\left\{\frac{[(n-1)c-n+2]^2-1}{[(n-1)c+1]^2} + \frac{[(n-m-1)c-n+m+2]^2[m+1-(n-m-1)c][m^2-mn+2m+2-(m+2)(n-m-1)c]}{4mc(n-m-1)[(n-m-1)^2c^2+(n-m-1)(2m-n+1)c-(m+1)(m-n+2)]}\right\},$$

$$\hat{\rho}(m) = \frac{[c(n-m-1)-(n-m-2)][m+1-c(n-m-1)]}{2cm(n-m-1)},$$

and τ *is the value of* c *such that* $\hat{\phi}(m) = 0$.

(c) *For* $c \geq \frac{n-m}{n-m-1}$,

$$\hat{\phi}(m) = \frac{1}{m+1}\left\{\frac{[(n-1)c-n+2]^2-1}{[(n-1)c+1]^2} - \hat{\rho}(m)\right\} \text{ and } \hat{\rho}(m) = \frac{mc-m-1}{mc}.$$

(ii) *If* $1 \leq m < \frac{n-1}{2}$, *we have:*

(a) *For* $c < \frac{m+1}{m}$,

$$\hat{\phi}(m) = \frac{[(n-1)c-n+2]^2-1}{(m+1)[(n-1)c+1]^2} \text{ and } \hat{\rho}(m) = 0.$$

(b) *For* $c \geq \frac{m+1}{m}$,

$$\hat{\phi}(m) = \frac{1}{m+1}\left\{\frac{[(n-1)c-n+2]^2-1}{[(n-1)c+1]^2} - \hat{\rho}(m)\right\} \text{ and } \hat{\rho}(m) = \frac{mc-m-1}{mc}.$$

Proof. Given $m+1$ firms in the coalition and $m \leq n-2$, there are $n-m-1 \geq 1$ firms not in the coalition. We consider two cases where these $n-m-1$ firms are active and inactive in equilibrium. By defining $\bar{\rho} \equiv \frac{mc-m-1}{mc}$, we know from Proposition 1 that the firms not in the coalition will be active (inactive) in equilibrium if $\rho > (\leq)\ \bar{\rho}$.

Case 1. Suppose that the NB solution has $\rho > \bar{\rho}$. We can rewrite (45) and (46) as

$$\pi_1^e(\phi, \rho|m) = F(\rho) + m\phi R \tag{50}$$

$$\pi_i^e(\phi, \rho|m) = G(\rho) - \phi R, \tag{51}$$

where

$$F(\rho) \equiv \left(1 - \frac{X^e(\rho|m)}{R}\right)^2 R + m\rho \left[1 - \frac{X^e(\rho|m)}{(1-\rho)R}\right] X^e(\rho|m) \tag{52}$$

$$G(\rho) \equiv \left[1 - \frac{X^e(\rho|m)}{(1-\rho)R}\right]^2 (1-\rho)R. \tag{53}$$

Note that both $\pi_1^e(\phi, \rho|m)$ and $\pi_i^e(\phi, \rho|m)$ are additively separable in terms of ϕ and ρ, and linear in ϕ. Hence, we solve the maximization problem for $W(\phi, \rho|m)$ firstly with respect to ϕ for a value of ρ given

$$W(\phi, \rho|m) = [F(\rho) + m\phi R - \pi_1^a] [G(\rho) - \phi R - \pi_i^a]^m . \tag{54}$$

Differentiating $W(\phi, \rho|m)$ with ϕ and considering the non-negativity restriction on ϕ, we have the following optimal ϕ for a given value of ρ:

$$\phi(\rho) = \max \left\{ \frac{1}{m+1} [G(\rho) - F(\rho) + \pi_1^a - \pi_i^a] , 0 \right\} \frac{1}{R}, \tag{55}$$

where the second-order condition for a maximum is met. Next, substituting *strictly positive* $\phi(\rho)R$ into $W(\phi, \rho|m)$, we have

$$W(\phi(\rho), \rho|m) = \frac{1}{(m+1)^{m+1}} [F(\rho) + mG(\rho) - \pi_1^a - m\pi_i^a]^{m+1} . \tag{56}$$

Note that the participation constraints for firms $1, ..., m+1$ guarantee non-negativity of the inside of the bracket in (56), and thus maximizing $W(\phi(\rho), \rho|m)$ is equivalent to maximizing the inside of the bracket in (56). The bargaining problem is then reduced to

$$\max_{\rho} F(\rho) + mG(\rho) - \pi_1^a(c) - m\pi_i^a. \tag{57}$$

From the first-order condition for a maximum, we obtain the critical value of ρ:

$$\rho^* = \frac{[c(n-m-1) - (n-m-2)][m+1-c(n-m-1)]}{2cm(n-m-1)}, \tag{58}$$

where the second-order condition for the maximum is met, and derive $\hat{\phi}(m)$ from (55).

We now check whether ρ^* meets the condition $\rho > \bar{\rho}$ so all firms are active in equilibrium. Since the values of ρ^* and $\bar{\rho}$ depend on n and m, we consider the following three possible subcases: $m = \frac{n-1}{2}$, $m < \frac{n-1}{2}$, and $m > \frac{n-1}{2}$.

Subcase 1.1. $m = \frac{n-1}{2}$. In this case, $\frac{m+1}{n-m-1} = \frac{m+1}{m} = \frac{n-m}{n-m-1}$ holds. If $c < \frac{n-m}{n-m-1}$, then we have $\rho^* > 0 > \bar{\rho}$. Since ρ^* meets the condition $\rho > \bar{\rho}$ and the non-negativity condition, $\hat{\rho}(m) = \rho^*$. If $c = \frac{n-m}{n-m-1}$, then we have $\rho^* = \bar{\rho} = 0$. If $c > \frac{n-m}{n-m-1}$, then we have $\bar{\rho} > 0 > \rho^*$. The zero value of ρ (the boundary value) cannot be the solution either, because $\bar{\rho} \geq 0$. Therefore, given $c \geq \frac{n-m}{n-m-1}$, a NB solution that makes all firms active in in equilibrium does not exist.

Subcase 1.2. $m > \frac{n-1}{2}$. In this case, $\frac{m+1}{m} < \frac{n-m}{n-m-1} < \frac{m+1}{n-m-1}$ holds. If $c < \frac{n-m}{n-m-1}$, then we have $\rho^* > \bar{\rho}$ and $\rho^* > 0$ so $\hat{\rho}(m) = \rho^*$. If $c \geq \frac{n-m}{n-m-1}$, then we have $\rho^* \leq \bar{\rho}$ and $\bar{\rho} > 0$, which means that neither ρ^* nor the zero value of ρ can be the solution.

Subcase 1.3. $m < \frac{n-1}{2}$. In this case, $\frac{m+1}{n-m-1} < \frac{n-m}{n-m-1} < \frac{m+1}{m}$ holds. If $c \leq \frac{m+1}{n-m-1}$, then we have $\rho^* \geq 0 > \bar{\rho}$ so $\hat{\rho}(m) = \rho^*$. However, this case can be ignored because $\frac{m+1}{n-m-1} < 1$ which implies $c < 1$. If $\frac{m+1}{n-m-1} < c < \frac{m+1}{m}$, then we have $\rho^* < 0$ and $\bar{\rho} < 0$, which means that the zero value of ρ can be the bargaining solution. If $c \geq \frac{m+1}{m}$, then we have $\rho^* < 0 \leq \bar{\rho}$. In this case, the zero value of ρ cannot be the solution. Hence, for $c \geq \frac{m+1}{m}$, the solution of ρ that makes all firms active doesn't exist.

The above analysis is under the presumption that $\phi(\rho)$ in (55) is strictly positive. If $\phi(\rho) = 0$ due to a small value of c, we can find the solution of ρ by solving the maximization problem of $W(\phi(\rho) = 0, \rho|m)$ by choosing ρ. In this case, we can derive $\hat{\rho}(m)$ by solving a 5th-degree polynomial numerically.

Case 2. Suppose that the NB solution has $\rho \leq \bar{\rho}$. This problem is akin to the case $m = n - 1$ because only the firms in the coalition are active in equilibrium. It follows that PC_1 and PC_i are linear and downward sloping, they intersect in the first quadrant, and PC_1 is flatter than PC_i. The feasible region of (ϕ, ρ) is the area bordered by PC_1, PC_i, the vertical axis, and the horizontal line $\rho = \bar{\rho}$. Like the case $m = n - 1$, any interior point in the feasible region cannot be the solution. Thus, the NB solution is either on the vertical axis ($\phi = 0$) or on the horizontal line ($\rho = \bar{\rho}$). Given $\phi = 0$, we find that the optimal ρ

that maximizes W is

$$\rho^0 = \frac{(n-1)^2(m+2)c^2 - 2(n-1)(mn-2m+n-3)c + mn^2 - m^2 - 4mn + n^2 + m - 4n + 3}{(m+2)[c(n-1)+1]^2}. \quad (59)$$

Given $\rho = \bar{\rho}$, we find that the optimal ϕ that maximizes W is

$$\bar{\phi} = \frac{1}{m+1}\left\{\frac{[(n-1)c-n+2]^2-1}{[(n-1)c+1]^2} - \bar{\rho}\right\}. \quad (60)$$

Since $W(\bar{\phi}, \bar{\rho}) > W(0, \rho^0)$ for any $m \leq n-2$, and $c > 1$, then $\hat{\phi}(m) = \bar{\phi}$ and $\hat{\rho}(m) = \bar{\rho}$.

Proposition 8 *Given $m \in \{1, ..., n-2\}$ determined in stage 1, the TIOLI solution uniquely exists in each of the following cases in stage 2.*

(i) If $\frac{n-1}{2} \leq m < n-1$, we have:

(a) For $c \leq \kappa$,

$$\tilde{\phi}(m) = 0 \text{ and } \tilde{\rho}(m) = \rho \text{ such that } \pi_i^e(0, \rho|m) = \pi_i^a.$$

(b) For $\kappa < c < \frac{n-m}{n-m-1}$,

$$\tilde{\phi}(m) = \phi^{**} \text{ and } \tilde{\rho}(m) = \frac{[c(n-m-1)-(n-m-2)][m+1-c(n-m-1)]}{2cm(n-m-1)},$$

where

$$\phi^{**} \equiv \frac{[(c - c\tilde{\rho}(m) - 1)(n-m-1) + 1 - \tilde{\rho}(m)]^2}{[(n-m-1)c+m+1]^2(1-\tilde{\rho}(m))} - \frac{1}{[(n-1)c+1]^2}$$

and κ is the value of c such that $\phi^{**} = 0$.

(c) For $c \geq \frac{n-m}{n-m-1}$,

$$\tilde{\phi}(m) = \phi^{**} \text{ and } \tilde{\rho}(m) = \frac{mc-m-1}{mc}.$$

(ii) If $1 \leq m < \frac{n-1}{2}$, we have:

(a) For $c < \frac{m+1}{m}$,

$$\tilde{\phi}(m) = \phi^{**} \text{ and } \tilde{\rho}(m) = 0.$$

(b) For $c \geq \frac{m+1}{m}$,

$$\tilde{\phi}(m) = \phi^{**} \text{ and } \tilde{\rho}(m) = \frac{mc - m - 1}{mc}.$$

Proof. Since we know from Proposition 1 that the firms not in the coalition will be active (inactive) in equilibrium if $\rho > (\leq)\ \bar{\rho}$, we consider the two cases below.

Case 1. Suppose that the TIOLI solution has $\rho > \bar{\rho}$. We rewrite π_1^e and π_i^e for $i = 2, ..., m+1$ as in (50) and (51) respectively. Since firm 1 chooses (ϕ, ρ) on PC_i to extract maximum surplus from each recipient firm, we have $\pi_i^e(\phi, \rho|m) = \pi_i^a$, i.e.,

$$\phi(\rho) = \max\{0, G(\rho) - \pi_i^a\} \frac{1}{R}. \tag{61}$$

Substituting *strictly positive* $\phi(\rho)$ into $\pi_1^e(\phi, \rho|m)$ in (50), we have

$$\pi_1^e(\phi, \rho|m) = F(\rho) + mG(\rho) - m\pi_i^a(c). \tag{62}$$

Since $F(\rho) + mG(\rho)$ is a common component in both (62) and (57), the optimal value of ρ for the TIOLI offer is the same as the value of ρ in the NB solution. Therefore, we have $\tilde{\rho}(m) = \rho^*$ as in (58) and derive $\tilde{\phi}(m)$ from (61).

The above analysis is under the presumption that $\phi(\rho)$ in (61) is strictly positive. If $\phi(\rho) = 0$ due to a small value of c, we can numerically derive the solution of ρ by solving for the value of ρ on PC_i given $\phi = 0$

Case 2. Suppose that the TIOLI solution has $\rho \leq \bar{\rho}$. The optimal value of ρ for the TIOLI solution making the $(n-m-1)$ firms not in the coalition inactive in stage 3 exists only when $m \geq \frac{n-1}{2}$ and $c \geq \frac{n-m}{n-m-1}$ *or* $m < \frac{n-1}{2}$ and $c \geq \frac{m+1}{m}$. Firm 1's problem becomes akin to the case $m = n-1$ because only the firms in the coalition are active in equilibrium. It follows that PC_i is linear and downward sloping, and that firm 1 chooses (ϕ, ρ) on PC_i such that $\rho = \bar{\rho}$.

A.4 Proof of Proposition 4

NB regime

Given $c \geq 2$, Propositions 2 and 3 imply that firm 1's expected payoffs can be written as:

$$\hat{\pi}_1(m) = \begin{cases} \frac{n(n-1)^2c^2-2(n-1)(n^2-3n+1)c+n^3-6n^2+8n-2}{n[c(n-1)+1]^2}R & \text{if } m = n-1 \\ \frac{1}{m+1}\left\{\frac{c-1}{c} + \frac{m[c(n-1)-n+2]^2-m}{[c(n-1)+1]^2}\right\}R & \text{if } 1 \leq m \leq n-2, \end{cases} \tag{63}$$

where $\hat{\pi}_1(m) \equiv \pi_1^e(\hat{\phi}(m), \hat{\rho}(m)|m)$.

Given $m = 1, ..., n-2$, we find that

$$\frac{\partial \hat{\pi}_1(m)}{\partial m} = \frac{1}{(m+1)^2}\left\{\frac{(n-1)^2 - 2[c(n-1)+1](n-1)}{[c(n-1)+1]^2} - \frac{1}{c}\right\} R < 0 \tag{64}$$

if and only if

$$f(c,n) \equiv 3c^2(n-1)^2 - c(n-1)(n-5) + 1 > 0. \tag{65}$$

Suppose $n \geq 5$. Since $f(2,5) > 0$, $\frac{\partial f(c,n)}{\partial c} > 0$, and $\frac{\partial f(c,n)}{\partial n} > 0$ for all $c \geq 2$ and $n \geq 5$, we have $\frac{\partial \hat{\pi}_1(m)}{\partial m} < 0$ given $c \geq 2$ and $n \geq 5$.

We also find that $\hat{\pi}_1(n-1) < \hat{\pi}_1(1)$ if and only if

$$h(c,n) \equiv (n-4)(n-1)^2c^2 - (n-2)(n^2-4n+2)c - n > 0. \tag{66}$$

Since $h(2,5) > 0$, $\frac{\partial h(c,n)}{\partial c} > 0$, and $\frac{\partial h(c,n)}{\partial n} > 0$ for all $c \geq 2$ and $n \geq 5$, we have $h(c,n) > 0$ given $c \geq 2$ and $n \geq 5$, i.e., $\hat{\pi}_1(n-1) < \hat{\pi}_1(1)$ for all $c \geq 2$ and $n \geq 5$. Therefore, firm 1's decision in stage 1 is $\hat{m}(n) = 1$ for $n \geq 5$.

Suppose $n = 4$. Since $f(2,4) > 0$ and $\frac{\partial f(c,4)}{\partial c} > 0$ for all $c \geq 2$, we have $\frac{\partial \hat{\pi}_1(m)}{\partial m} < 0$ for given $c \geq 2$ and $m \leq 2$. We also have $h(c,4) < 0$ for all $c \geq 2$, which means that $\hat{\pi}_1(3) > \hat{\pi}_1(1)$. Thus, $\hat{m}(4) = 3$.

Finally, suppose $n = 3$. Since $h(c,3) < 0$ for all $c \geq 2$, we have $\hat{\pi}_1(2) > \hat{\pi}_1(1)$. So, $\hat{m}(3) = 2$.

TIOLI regime

Given $c \geq 2$, Proposition 2 implies that firm 1's expected payoffs can be written as:

$$\tilde{\pi}_1(m) = \begin{cases} \frac{(n-1)^2c^2+2(n-1)c-n^2+2}{[c(n-1)+1]^2}R & \text{if } m = n-1 \\ \left\{\frac{c-1}{c} - \frac{m}{[c(n-1)+1]^2}\right\}R & \text{if } 0 \leq m \leq m-2, \end{cases} \tag{67}$$

where $\tilde{\pi}_1(m) \equiv \pi_1^e(\tilde{\phi}(m), \tilde{\rho}(m)|m)$.

Given $m = 1, ..., n-2$, we find that

$$\frac{\partial \tilde{\pi}_1(m)}{\partial m} = -\frac{1}{[c(n-1)+1]^2}R < 0. \tag{68}$$

We also find that $\tilde{\pi}_1(n-1) > \tilde{\pi}_1(1)$ if and only if

$$g(c,n) \equiv c^2(n-1)^2 - cn(n-2) + 1 > 0. \tag{69}$$

Since $g(2,3) > 0$, $\frac{\partial g(c,n)}{\partial c} > 0$, and $\frac{\partial g(c,n)}{\partial n} > 0$ for all $c \geq 2$ and $n \geq 3$, we have $g(c,n) > 0$, i.e., $\tilde{\pi}_1(n-1) > \tilde{\pi}_1(1)$. Firm 1 thus chooses $\tilde{m}(n) = n-1$.

A.5 Proof of Proposition 5

NB regime with $n = 3$

According to the NB solutions derived in Proposition 2 and in the proof of Proposition 3, we have

$$\left(\hat{\phi}(m), \hat{\rho}(m)\right) = \begin{cases} (0, \rho^*(1)) & \text{if } m = 1 \text{ and } c \leq 1.5334 \\ \left(\frac{12c^4-4c^3-29c^2+8c+4}{8c(2c+1)^2}, \frac{2-c}{2}\right) & \text{if } m = 1 \text{ and } c > 1.5334 \\ \left(0, \frac{2(c-1)}{2c+1}\right) & \text{if } m = 2, \end{cases} \tag{70}$$

where $\rho^*(1) \equiv arg\max_\rho W(0,\rho|1)$. Given the above license fees, firm 1's expected payoff can be written as

$$\hat{\pi}_1(m) = \begin{cases} \frac{(2c-1)^2 R}{(2c+1)^2} & \text{if } m = 0 \\ \pi_1^e(0,\rho^*(1)|1) & \text{if } m = 1 \text{ and } c \leq 1.5334 \\ \frac{c(4c^2+20c-15)R}{8(2c+1)^2} & \text{if } m = 1 \text{ and } c > 1.5334 \\ \frac{(6c-5)R}{3(2c+1)} & \text{if } m = 2. \end{cases} \tag{71}$$

The top left panel of Figure 7 shows each firm 1's expected payoff under NB for $m \in \{0,1,2\}$. Since $\hat{\pi}_1(2) > \hat{\pi}_1(m)$ for $m \in \{0,1\}$ and for any given $c \in (1,2)$, then $\hat{m}(3) = 2$ in stage 1.

NB regime with $n = 4$

According to the NB solutions derived in Proposition 2 and in the proof of Proposition 3, we have

$$\left(\hat{\phi}(m),\hat{\rho}(m)\right) = \begin{cases} \left(\frac{3(3c-1)(c-1)}{2(3c+1)^2}, 0\right) & \text{if } m = 1 \\ (0,\rho^*(2)) & \text{if } m = 2 \text{ and } c < 1.8551 \\ \left(\frac{18c^4-15c^3-25c^2-25c+15}{12(c+1)(3c+1)^2}, \frac{3-c}{4}\right) & \text{if } m = 2 \text{ and } c > 1.8551 \\ \left(0, \frac{3(c-1)(15c+1)}{5(3c+1)^2}\right) & \text{if } m = 3, \end{cases} \tag{72}$$

where $\rho^*(2) \equiv arg\max_\rho W(0,\rho|2)$. Given the above license fees, firm 1's expected payoff can be written as

$$\hat{\pi}_1(m) = \begin{cases} \frac{(3c-2)^2 R}{(3c+1)^2} & \text{if } m = 0 \\ \frac{(54c^4-35c^2-10c+7)R}{4(c+1)^2(3c+1)^2} & \text{if } m = 1 \\ \pi_1^e(0,\rho^*(2)|2) & \text{if } m = 2 \text{ and } c < 1.8551 \\ \frac{(9c^3+78c^2-95c+24)R}{12(3c+1)^2} & \text{if } m = 2 \text{ and } c > 1.8551 \\ \frac{(18c^2-15c-1)R}{2(3c+1)^2} & \text{if } m = 3. \end{cases} \tag{73}$$

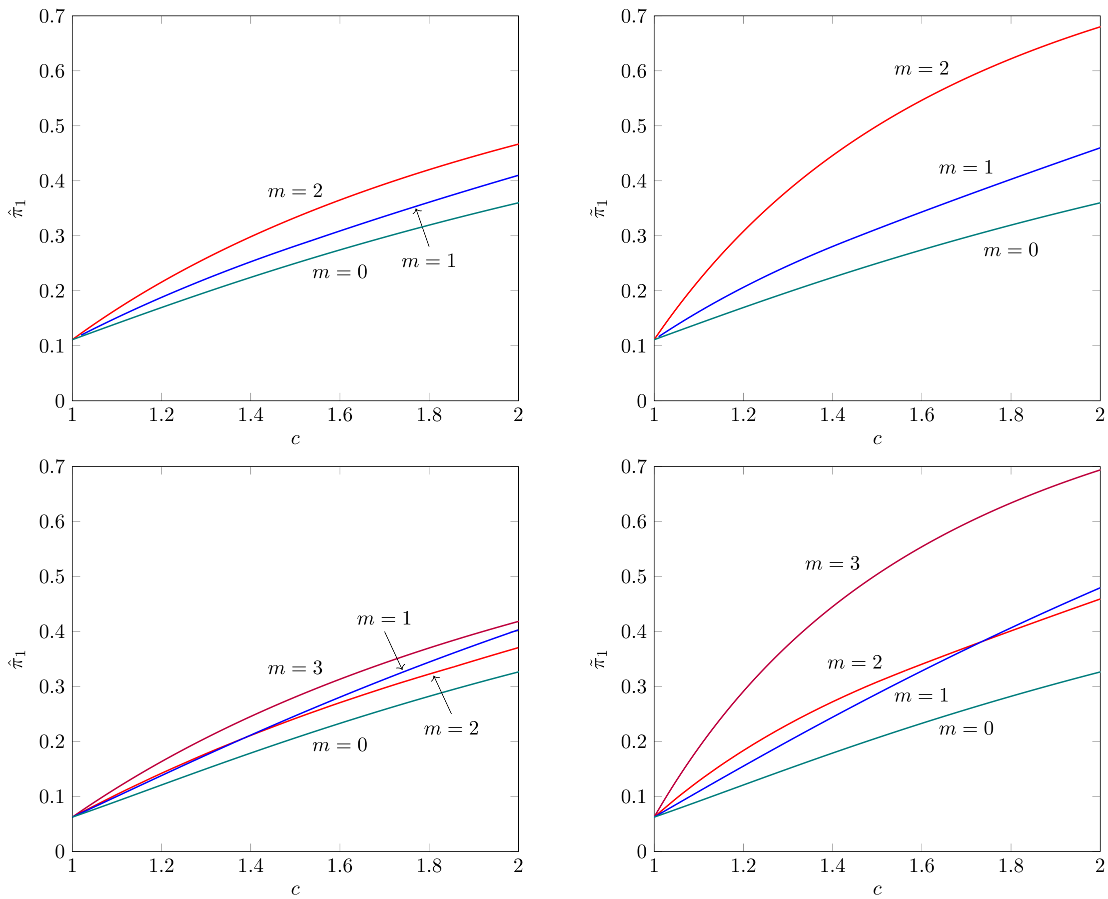


Figure 7: Firm 1's expected payoff in equilibrium under NB (left) and TIOLI (right) for $n = 3$ (top) and $n = 4$ (bottom).

The bottom left panel of Figure 7 shows each firm 1's expected payoff under NB for $m \in \{0, 1, 2, 3\}$. Since $\hat{\pi}_1(3) > \hat{\pi}_1(m)$ for $m \in \{0, 1, 2\}$ and for any given $c \in (1, 2)$, then $\hat{m}(3) = 3$ in stage 1.

TIOLI regime with $n = 3$

According to the TIOLI solutions derived in Proposition 2 and in the proof of Proposition 3, we have

$$\left(\hat{\phi}(m), \hat{\rho}(m)\right) = \begin{cases} (0, \rho^{**}(1)) & \text{if } m = 1 \text{ and } c \leq 1.4376 \\ \left(\frac{(2c-1)(2c^3-c^2-2c-1)}{2c(2c+1)^2}, \frac{2-c}{2}\right) & \text{if } m = 1 \text{ and } c > 1.4376 \\ \left(0, \frac{4(c+2)(c-1)}{(2c+1)^2}\right) & \text{if } m = 2, \end{cases} \tag{74}$$

where $\rho^{**}(1) \equiv \rho$ such that $\pi_2^e(0, \rho|1) = \pi_2^a$. Given the above license fees, firm 1's expected payoff can be written as

$$\tilde{\pi}_1(m) = \begin{cases} \frac{(2c-1)^2 R}{(2c+1)^2} & \text{if } m = 0 \\ \pi_1^e(0, \rho^{**}(1)|1) & \text{if } m = 1 \text{ and } c \leq 1.4376 \\ \frac{[c(2c+1)^2-4]R}{4(2c+1)^2} & \text{if } m = 1 \text{ and } 1.4376 < c < 2 \\ \frac{(4c^2+4c-7)R}{(2c+1)^2} & \text{if } m = 2. \end{cases} \tag{75}$$

The top right panel of Figure 7 shows each firm 1's expected payoff under TIOLI for $m \in \{0, 1, 2\}$. Since $\tilde{\pi}_1(2) > \tilde{\pi}_1(m)$ for $m \in \{0, 1\}$ and for any given $c \in (1, 2)$, then $\tilde{m}(3) = 2$ in stage 1.

TIOLI regime with $n = 4$

According to the NB solutions derived in Proposition 2 and in the proof of Proposition 3, we have

$$\left(\tilde{\phi}(m), \hat{\rho}(m)\right) = \begin{cases} \left(\frac{3(c-1)(c+1)(3c^2+2c+3)}{(c+3)^2(3c+1)^2}, 0\right) & \text{if } m = 1 \\ (0, \rho^{**}(2)) & \text{if } m = 2 \text{ and } c < 1.5628 \\ \left(\frac{9c^4-12c^3-2c^2-3}{4(3c+1)^2(c+1)}, \frac{3-c}{4}\right) & \text{if } m = 2 \text{ and } c > 1.5628 \\ \left(0, \frac{3(c-1)(3c+5)}{(3c+1)^2}\right) & \text{if } m = 3, \end{cases} \tag{76}$$

where $\rho^{**}(2) \equiv \rho$ such that $\pi_i^e(0, \rho|2) = \pi_i^a$ for $i = 2, 3$. Given the above license fees, firm 1's expected payoff can be written as

$$\tilde{\pi}_1(m) = \begin{cases} \frac{(3c-2)^2 R}{(3c+1)^2} & \text{if } m = 0 \\ \frac{(36c^4-12c^3-13c^2-2c-1)R}{2(c+1)^2(3c+1)^2} & \text{if } m = 1 \\ \pi_1^e(0, \rho^{**}(2)|2) & \text{if } m = 2 \text{ and } c < 1.5628 \\ \frac{(9c^3+6c^2+c-8)R}{4(3c+1)^2} & \text{if } m = 2 \text{ and } c > 1.5628 \\ \frac{(9c^2+6c-14)R}{(3c+1)^2} & \text{if } m = 3. \end{cases} \tag{77}$$

The bottom right panel of Figure 7 shows each firm 1's expected payoff under NB for $m \in \{0, 1, 2, 3\}$. Since $\tilde{\pi}_1(3) > \tilde{\pi}_1(m)$ for $m \in \{0, 1, 2\}$ and for any given $c \in (1, 2)$, then $\tilde{m}(3) = 3$ in stage 1.

A.6 Proof of Proposition 6

Case 1: $c \geq 2$

Under NB, Proposition 4 states that $\hat{m}(n) = 1$. That is, firm 1 shares its technology only with firm 2 and all other firms do not participate in the contest. Firm 2's expected payoff in equilibrium can be written as

$$\hat{\pi}_2(1) = \frac{1}{2}\left\{\frac{c-1}{c} - \frac{[c(n-1)-n+2]^2 - 1}{[c(n-1)+1]^2}\right\} R. \tag{78}$$

Since $\hat{\pi}_2(1) > \pi_2^a$, the equilibrium network structure $\hat{m}(n) = 1$ is pairwise stable under NB.

Under TIOLI, Proposition 4 states that $\tilde{m}(n) = n - 1$. That is, firm 1 shares its technology with all other firms and every firm participates in the contest. Firm i's expected payoff in equilibrium, for $i = 2, ..., n$, can be written as

$$\tilde{\pi}_i(n-1) = \frac{1}{[(n-1)c+1]^2} R = \pi_i^a. \tag{79}$$

Since firm 1's technology is radical, $\tilde{\pi}_n(n-2) = 0 < \pi_n^a$ and, therefore, the equilibrium network structure $\tilde{m}(n) = n - 1$ is pairwise stable under TIOLI.

Case 2: $c \in (1, 2)$ **and** $n \in \{3, 4\}$

Under either NB or TIOLI, Proposition 5 states that $\hat{m}(n) = \tilde{m}(n) = n - 1$.

Given $n = 3$, we find that under NB

$$\hat{\pi}_3(m) = \begin{cases} \pi_3^e(0, \hat{\rho}(1)|1) & \text{if } m = 1 \text{ and } 1 < c \leq 1.5334 \\ \frac{(2-c)^2 R}{4} & \text{if } m = 1 \text{ and } 1.5334 < c < 2 \\ \frac{R}{3(2c+1)} & \text{if } m = 2. \end{cases} \tag{80}$$

As shown in the top left panel of Figure 8, $\hat{\pi}_3(2) > \hat{\pi}_3(1)$ for all $c \in (1, 2)$ so firm 3 does not have an incentive to leave the coalition.

Given $n = 4$, we find that under NB

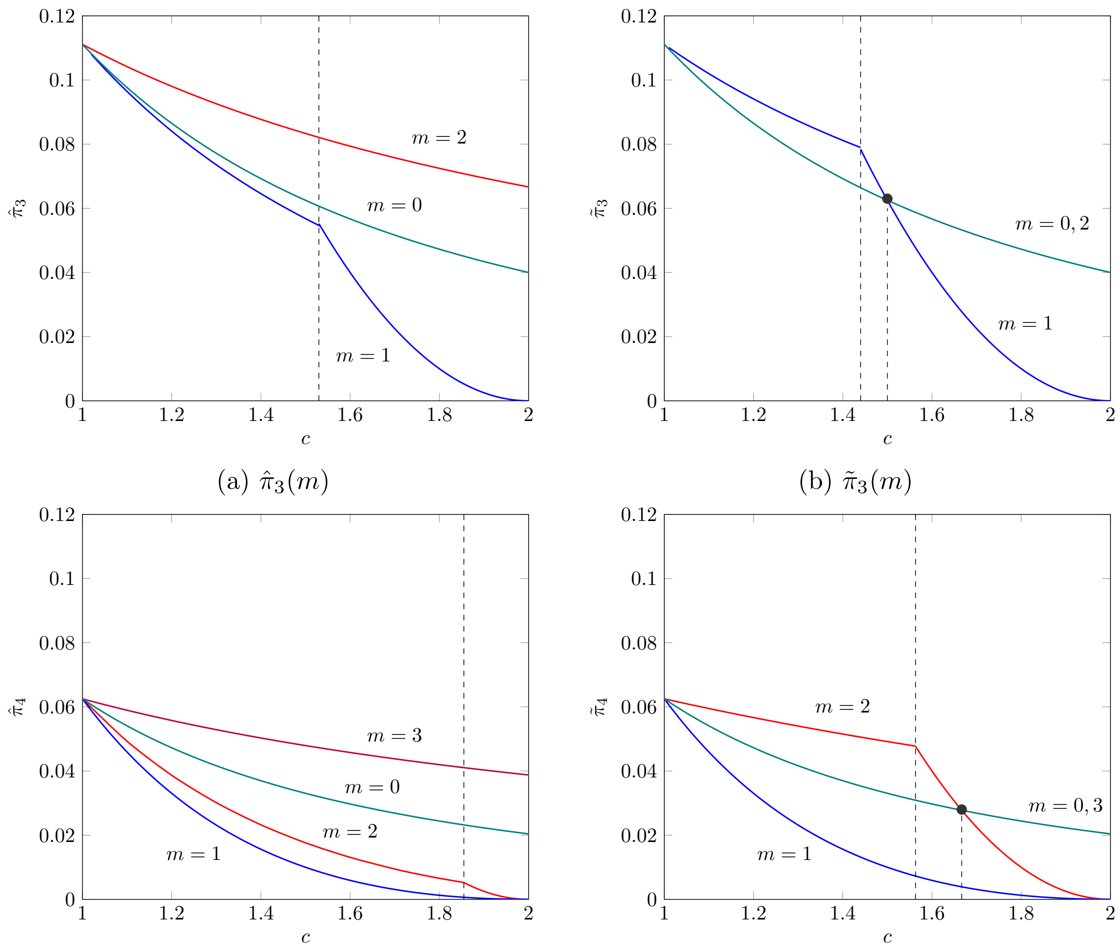
$$\hat{\pi}_4(m) \begin{cases} \pi_4^e(0, \hat{\rho}(2)|2) & \text{if } m = 2 \text{ and } 1 < c \leq 1.8551 \\ \frac{(2-c)^2 R}{4} & \text{if } m = 2 \text{ and } 1.8551 < c < 2 \\ \frac{(9c+1)R}{10(3c+1)^2} & \text{if } m = 3. \end{cases} \tag{81}$$

As shown in the bottom left panel of Figure 8, $\hat{\pi}_4(3) > \hat{\pi}_4(2)$ for all $c \in (1, 2)$ so firm 4 does not have an incentive to leave the coalition.

Given $n = 3$, we find that under TIOLI

$$\tilde{\pi}_3(m) = \begin{cases} \pi_3^e(0, \tilde{\rho}(1)|1) & \text{if } m = 1 \text{ and } 1 < c \leq 1.4376 \\ \frac{(2-c)^2 R}{4} & \text{if } m = 1 \text{ and } 1.4376 < c < 2 \\ \frac{R}{(2c+1)^2} & \text{if } m = 2. \end{cases} \tag{82}$$

As shown in the top right panel of Figure 8, $\tilde{\pi}_3(2) > \tilde{\pi}_3(1)$ (i.e., firm 3 does not have an incentive to leave the coalition) if and only if $c \in (1.5, 2)$.



(a) $\hat{\pi}_3(m)$

(b) $\tilde{\pi}_3(m)$

(c) $\hat{\pi}_4(m)$

(d) $\tilde{\pi}_4(m)$

Figure 8: Firm n's expected payoff in equilibrium under NB (left) and TIOLI (right) for $n = 3$ (top) and $n = 4$ (bottom).

Given $n = 4$, we find that under TIOLI

$$\tilde{\pi}_4(m) = \begin{cases} \pi_4^e(0, \tilde{\rho}(2)|1) & \text{if } m = 2 \text{ and } 1 < c \leq 1.5628 \\ \frac{(2-c)^2 R}{4} & \text{if } m = 2 \text{ and } 1.5628 < c < 2 \\ \frac{R}{(3c+1)^2} & \text{if } m = 3. \end{cases} \tag{83}$$

As shown in the top right panel of Figure 8, $\tilde{\pi}_4(3) > \tilde{\pi}_4(2)$ (i.e., firm 4 does not have an incentive to leave the coalition) if and only if $c \in (1.6667, 2)$.